

Condensation Irrigation

A Combined System for Desalination and Irrigation

Jenny Lindblom

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a combined system for desalination and irrigation

Doctoral Thesis

Jenny Lindblom

Division of Architecture and Water
Department of Civil, Environmental and Natural resources engineering
Luleå University of Technology
SE-97187 Luleå
Sweden

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Summary

Condensation Irrigation (CI) is a new irrigation method that combines desalination and subsurface irrigation by making use of saline water for supplying clean irrigation. In this system, solar stills are used for evaporating non-potable water, and the formed vapour heats and humidifies the ambient air above the water surface. The warm, humid air is led from the stills into a system of horizontally buried drainage pipes. While flowing through the pipes, the air is cooled by the ground and vapour precipitates as freshwater inside the pipes. The perforations in the pipe wall enable the formed freshwater to percolate into the surrounding soil, and thereby irrigate it. Some of the humid air also infiltrates the soil through the perforations, which further increases irrigation by vapour condensation in the cooler ground. The airflow through the ground supplies soil aeration, which is important for high crop yield.

Because the CI system generates freshwater from saline or otherwise contaminated water sources, this system can operate in locations that would normally lack irrigation possibilities. This subsurface irrigation system has also further advantages, such as reduced water losses through surface evaporation and deep percolation, increased soil aeration, and low tech / low cost design.

To investigate the potential of the CI system, an implicit transient finite element simulation model, CI2D, was developed in Matlab, that was able to simulate the complex coupled mechanisms of gas, liquid and heat transfer in the soil-pipe system, including water evaporation and condensation. The validated and verified model also included solar radiation, root water extraction, and surface evaporation.

The CI2D model was used to simulate a reference example of a theoretical CI facility in Malta. The irrigation rate under steady operation was 3.44 mm d^{-1} and the root water uptake was 19.8% of the supplied water. By lowering the inlet air temperature, the crop could be placed closer to the pipes without the roots being overheated. The irrigation rate obtained by decreasing the inlet air temperature from 70°C to 50°C , and reducing the pipe spacing from 1.2 m to 0.6 m, was 3.00 mm d^{-1} . The root water uptake was, however, increased to 48% of the irrigation, resulting in a higher root water uptake.

The principle behind CI can be used for drinking water production by using pipes without perforations in the ground. The condensed freshwater can then be collected at the pipe endings. This system was simulated under the same reference scenario as the irrigation system. The daily water production rate in a 50 m long pipe was in the example 135 kg d^{-1} , corresponding to 2.26 mm d^{-1} .

A small scale laboratory setup where humid air was led through a perforated pipe in a sand box was tested and theoretically simulated. In the experiments, the importance of a free flow path for the gas phase through the soil was visualized. It could therefore be concluded that the CI system should not be implemented in low-permeability soils. From simulations in CI2D, it was evident that soils

with high capillarity are unsuitable for CI systems as well, because the water accumulation around the pipe prevents humid air from entering the soil through the perforations.

CI is a system with many unexplored possible designs and applications. For example, by leading the cooler saline feed water to the solar stills through the perforated irrigation pipes, the vapour condensation in the pipes would increase. This would also increase the solar still efficiency since the incoming saline water would be preheated by the humid airflow. In future work on this system, this, and other suggested improvements should be explored.

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Luleå, November 2012

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List of Papers

Paper I Lindblom, J., Nordell, B. (2006). Water production by underground condensation of humid air. Desalination 2006;189:248–260

Paper II Lindblom, J., Nordell, B. (2007). Underground production of Humid air for drinking water production and subsurface irrigation. Desalination 2007;203:417–434

Paper III Lindblom, J., Frishfelds, V., Lundström, T.S. (2012). Modeling Non-isothermal multiphase transport in porous media using the modified Picard method (submitted for publication)

Paper IV Lindblom, J., Nordell, B. (2012). Condensation Irrigation - Operation simulation and feasibility study (submitted for publication)

Paper V Lindblom, J., Nordell, B. (2011). Experimental study of underground irrigation by condensation of humid air in perforated pipes. Technical report. Division of Architecture and Water, Department of Civil, Environmental and Natural resources engineering, Luleå University of Technology. ISBN 978-91-7439-513-6

Appendix I Discretization of mass and heat balance equations in the simulation program CI2D.

Papers Not Included in Thesis

- Lindblom, J., Nordell, B. (2003) Condensation Irrigation - a system for desalination and irrigation. Proceedings at Futurestock, 2003. Warsaw, Poland.
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- Lindblom, J., Nordell, B. (2006) Condensation Irrigation - a combined system for desalination & irrigation. Proceedings at WREEC, 2006. Tripoli, Libya.
- Lindblom, J., Nordell, B. (2006) Condensation Irrigation - a solar driven system for irrigation and drinking water production. Proceedings at WREC IX, 2006. Florence, Italy.
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1

Introduction

Water scarcity is an increasing problem in the arid and semiarid parts of the world. Along with the dramatic population growth and improved living standards, existing freshwater supplies are deteriorating (Kuylenstierna *et al.* (2009); Fritzmann *et al.* (2007); El-Kady and El-Shibini (2001)). When it comes to dividing the existing freshwater resource between industrial, domestic, and agricultural sectors, it often falls on agriculture to give up water resources for the benefit of other sectors (Beltran (1999); Kandelous and Simunek (2010)). The freshwater needed for irrigation is therefore frequently replaced by marginal quality water, such as brackish groundwater, waste or drainage water (Qadir *et al.* (2007)). The result of using marginal quality water for irrigation is degradation of the farmland (Pereira *et al.* (2002)).

The only way to produce more freshwater is by desalinating seawater. This was known by early civilizations, and Greek seafarers from 200 AD used seawater for producing freshwater on long voyages by boiling seawater in brass vessels. A sea sponge was placed over the opening of the vessel, so that the rising steam was collected in the sponge. By squeezing the sponge, freshwater was collected (Kalogirou (2005)).

The condensation irrigation (CI) system proposed in this thesis is a combined solar desalination and subsurface irrigation system, suited for semiarid, arid, and isolated sunny locations, where freshwater for irrigation is scarce, but saline water from either the sea or the groundwater is available. In the desalination part of the system, the saline water is heated inside solar stills and thus evaporates. The formed vapour heats and humidifies ambient air flowing above the water surface inside the still. The warm and humid air is led from the still into the subsurface irrigation system, comprising horizontal underground drainage pipes. While flowing through the pipes, the ground cools the humid air so that vapour condenses as freshwater inside the pipes. The perforations in the pipe wall enable

the water to percolate into the surrounding soil, and thereby irrigate it. Some of the humid air also infiltrates the soil through the perforations and contributes to soil aeration and irrigation by vapour condensation in the ground (Lindblom and Nordell (2012, 2007); Lindblom (2006)).

The CI technology can also be applied to drinking water production by using non-perforated pipes in the ground. The condensed water from the humid airflow can then be collected at the pipe endings by applying a small tilt to the buried pipes (Lindblom and Nordell (2012, 2006)).

The ideas behind the CI system were first developed at Luleå University of Technology (LTU) in the mid 1980's, and initial studies on the subject were carried out as a series of Masters Theses (Widegren (1986); Göhlman (1987); Gustafsson and Lindblom (2001)). In a project connected to the theories of CI, a greenhouse climate system was developed and implemented at the Hietala Market gardens in Övertorneå, Sweden (Nordell (1987)).

Independently of these studies, the Swiss company Ingenieurbüro Ruess und Hausherr constructed a CI plant in which seawater evaporated inside plastic tubes and was thereafter condensed in buried drainage pipes. A reported 50% reduction in the water consumption of tomato plants was observed in the system (Hausherr and Ruess (1993)). Other experiments on CI have been carried out by the National Research Institute for Agricultural Engineering in Tunisia (Lindblom (2006)).

1.1 Objectives

The construction of a CI system requires knowledge and understanding of the processes governing the water production and irrigation efficiency. Although the principle behind condensation irrigation is very simple, the theory involved is complex. To correctly anticipate the heat and mass transfer in the ground, transport of air, vapour, water, and heat, including phase changes, must be taken into account.

The main objectives of this thesis have been to develop an understanding of the CI system to a point where it would be possible to perform reliable design simulations of such systems. This thesis included the following steps:

1. Study the physics relating to mass and heat transfer in the soil and irrigation pipes
2. Develop a model for simulating CI systems, based on relevant physics
3. Determine the irrigation potential of CI systems
4. Analyze the specific factors influencing the irrigation yield in CI

To gain practical insight into how water, humid air, and heat spread through a soil during condensation irrigation, a small scale experiment was also built and analyzed.

1.2 Scope

In performed studies on condensation irrigation, the main focus has been on mass and heat transfer in the soil-pipe system. The influence of crop root water uptake, water exchange at the soil surface due to solar radiation, wind, etc. have to some extent also been examined. Detailed studies on pipe condensation have been left to future work.

The manner in which the air is heated and humidified prior to entering the subsurface irrigation pipes greatly affect the efficiency and economical feasibility of a condensation irrigation system. This part of the system has nevertheless been disregarded in this work.

2

Solar Desalination in Agriculture

Since the dawn of the industrial revolution, the global population has escalated, with increasing food and water requirements as a consequence. As the arid countries around the Arabian Gulf and North Africa started experiencing water scarcity some 50 years ago, seawater desalination became increasingly important to support the growing population with water for domestic, industrial, and agricultural use (Fritzmann *et al.* (2007); Greenlee *et al.* (2009)).

Due to the vast oil reserves found in these countries, the natural source for powering these large scale desalination plants was oil. Even now, less than 1% of the desalination plants in the world are powered by renewable energy (Garcia-Rodriguez (2002)).

Currently, 90-95% of the freshwater supply in the Gulf countries originates from seawater desalination plants powered by fossil fuel (El-Kady and El-Shibini (2001)). In the short time before these oil reserves are depleted, there must, however, be an energy shift towards more sustainable energy sources (Inoue *et al.* (2006)).

More countries are now experiencing water scarcity, many of them in sunny, arid regions lacking oil resources. The short term solution has so far been to import food and water (El-Kady and El-Shibini (2001)). It has, however, been estimated that up to 50% of the freshwater need of the rural population in these countries could be covered by small to medium scale solar desalination plants (Chaibi (2011)).

The agricultural sector is by far the largest freshwater consumer, withdrawing 70-75% of the world's available freshwater (Calzadilla *et al.* (2010)). In arid countries, irrigation in agriculture can account for more than 90% of the freshwater consumption. Because irrigation is vital for food security in arid regions, water efficient irrigation methods, such as drip or subsurface irrigation, could liberate water to further increase water and food security (Kuylenstierna *et al.* (2009)).

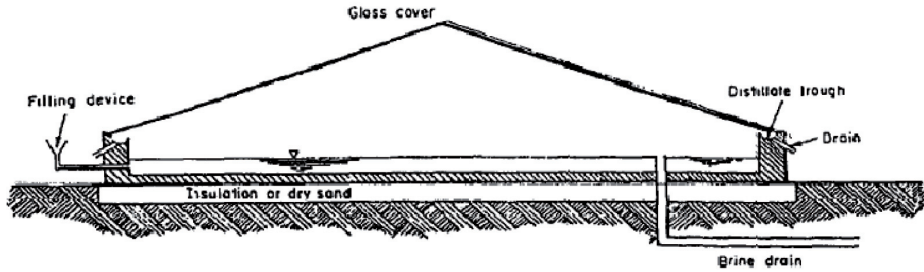


Figure 2.1: Principle outline of a basic solar still (Fath (1998))

Still, these are only used on 1% of the irrigation sites (Postel *et al.* (2001); Kalo-girou (2005)).

Another option for increasing irrigation would be to use desalination plants for producing safe irrigation water. Desalination is however a very energy intensive process, making the freshwater produced by fossil fuels an expensive resource in developing countries. Desalinated water is therefore only used in agriculture when growing high-value crops for example in greenhouses, since the irrigation demand in a modern greenhouse can be reduced by 60-90% of the irrigation demand on open fields (Chaibi (2003); Radhwan (2004)).

2.1 Solar Stills

Solar desalination is humankind's way to emulate nature's hydrological cycle: solar thermal energy is used to evaporate saline water, usually seawater, and as the vapour condenses, it is collected as freshwater.

The very first large-scale solar desalination plant was built in Chile in 1827 by the Swedish engineer Carlos Wilson. The plant consisted of 64 solar stills, which together produced 22.7 m^3 freshwater per day, corresponding to 4.9 kg m^{-2} . The solar desalination plant was operating successfully for 40 years (Arjunan *et al.* (2009)).

A solar still consists of an insulated water container with a transparent cover. Solar radiation transmitted through the cover is absorbed in black-painted bottom and sides of the container, and saline water evaporates due to the solar heat. The vapour rises in the air above the saline water surface, condenses on the cooler cover material, and forms droplets that run down into a gutter and into a storage tank (Fig. 2.1).

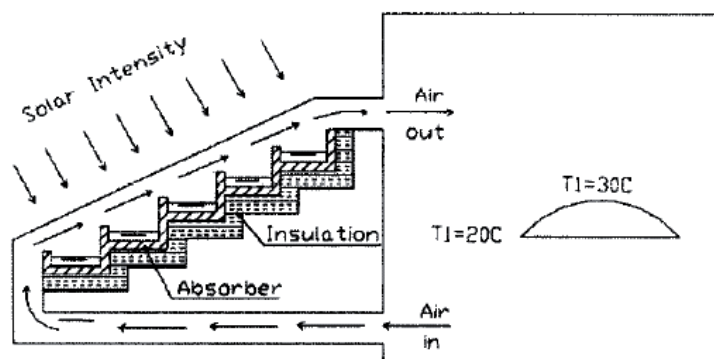


Figure 2.2: Stepped solar still connected to a greenhouse for heating and humidifying air (Radhwan (2004))

Solar stills are passive devices driven by free solar energy. The environmental impact and operating cost are insignificant, but the low production rates and large land area requirement makes the technology only suitable for small to medium scale production (Ayoub and Malaeb (2012); Kunze (2001)). Commercial passive solar stills produce about $3\text{--}4 \text{ kg m}^{-2} \text{ d}^{-1}$ (Kalogirou (1997)), but new innovative designs are able to increase this rate somewhat (Arjunan *et al.* (2009)).

In arid regions with high solar potential and large disposable land areas, solar stills could provide solar desalinated water for agriculture and domestic use at a very low operating cost.

2.2 Solar Stills Connected to Greenhouses

A stepped solar still connected to a greenhouse in Saudi Arabia was theoretically studied by Radhwan (2004). The still consisted of five trays of saline water in an insulated basin with an inclined glazing. Air from the greenhouse entered the solar still at the lowest point, flowed upwards over the trays to pick up vapour, and re-entered the greenhouse from the top of the still as vapour saturated air.

The estimated daily water yield from the still was about 4.9 kg m^{-2} , of which 0.8 kg m^{-2} was condensed on the glass cover, and 4.1 kg m^{-2} entered the greenhouse as vapour. The air temperature leaving the solar still reached 42°C during midday.

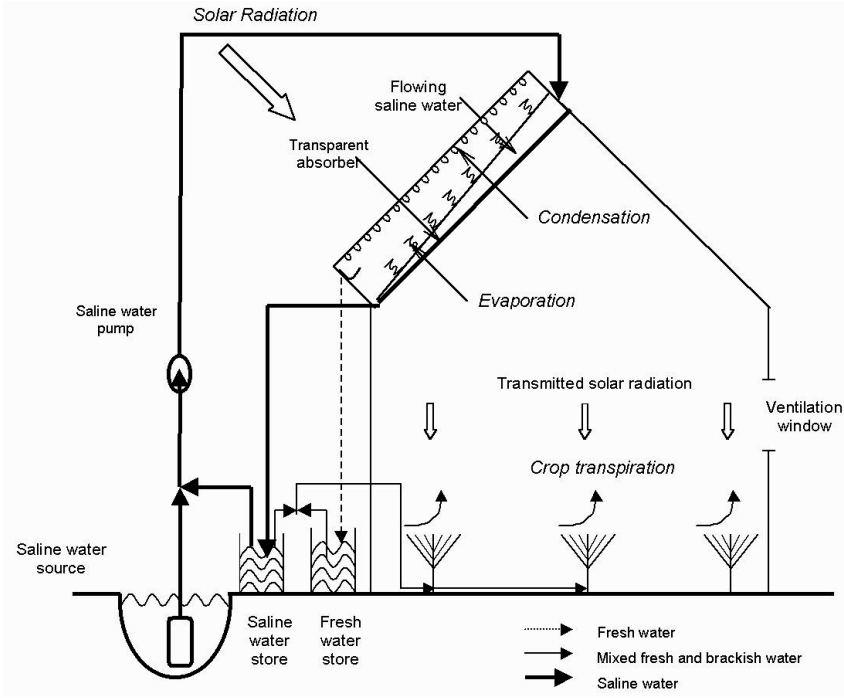


Figure 2.3: Greenhouse with integrated solar still on the roof (Chaibi (2003))

2.3 Solar Stills Integrated with Greenhouses

A roof-integrated water desalination system for greenhouses was built and analyzed by Chaibi (2003). The solar still consisted of a double glazed part of the south facing roof, in which the lower glass was only partly light transparent. Some of the incident solar radiation was absorbed in this glass layer, which then heated and evaporated saline water flowing in a thin layer between the glass sheets. The formed vapour condensed on the top glass cover and ran down into a freshwater store.

The produced water was used for irrigation in the greenhouse by mixing it with some brackish water to economize the freshwater supply.

Because a large fraction of the thermal radiation from the sun was absorbed in the glass, the temperature inside the greenhouse was reduced, which reduced the ventilation requirement.

When mixing the produced freshwater with the same amount of brackish water, the water production was enough to meet the water demand of low-canopy crops in a climate such as Tunisia (Chaibi and Jilar (2004)).

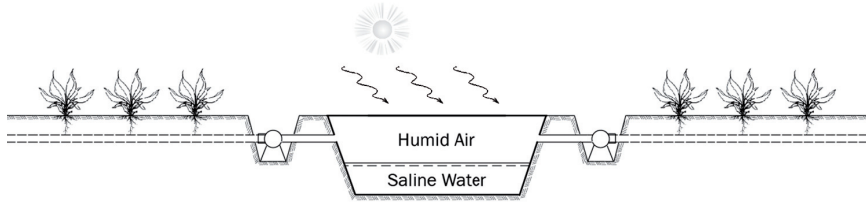


Figure 2.4: Principle outline of a CI system, showing a solar still with buried irrigation pipes extending in two directions.

2.4 Condensation Irrigation

The Condensation Irrigation (CI) system presented in this thesis uses solar thermal energy to evaporate saline, or otherwise polluted water, in solar stills. Ambient air is humidified by the warm water inside the still and thereafter led into an underground pipe system where it is cooled and the vapour precipitates as freshwater on the pipe walls.

If the buried pipes are perforated, condensed water and some humid air will percolate through the perforations into the ground and thereby irrigate and aerate the soil close to the rooting zone.

By using non-perforated pipes the condensed water from the pipes can be collected at the pipe endings and used for drinking or other purposes. The principle of the CI system is shown in Fig. 2.4.

As the airflow is dehumidified in the pipes, the surrounding soil gradually becomes heated, thereby reducing the water production efficiency. To decrease the ground temperature, cold ambient air is circulated through the pipe system at night.

The irrigation yield of the CI system depends on parameters such as pipe configuration, climate, soil type, and achievable inlet humid airflow properties (Kandelous and Simunek (2010); Lindblom (2006)). Water demand, rooting depth, and temperature sensitivity of the crop must also be taken into account. In the studies presented here, temperatures up to 32°C are considered stimulating for root growth (Arai-Sanoh *et al.* (2010)), while higher temperatures risk reducing root development and respiration (Rachmilevitch *et al.* (2006)). In the CI system, the pipe configuration should therefore be designed so that this critical temperature is reached at the pipe walls, but never in the rooting zone in the soil. Plant roots are then free to develop in the soil between the irrigation pipes, but will avoid growing into the pipes, where they would block the airflow. Root intrusion

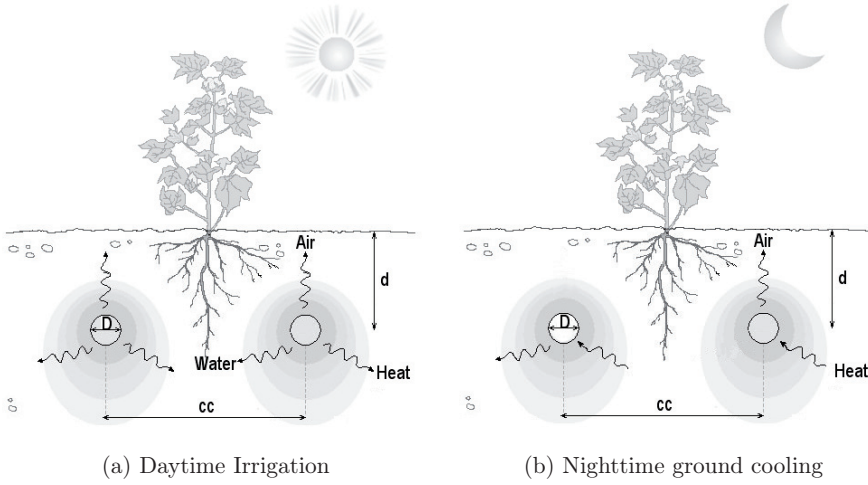


Figure 2.5: Section showing two buried drainage pipes buried on either side of cultivated crop. During the day the pipes supply water, humid air, and heat to the ground. At night, the ground is cooled by night air flowing through the pipe and soil. The pipe depth, diameter and spacing are denoted by d (m), D (m), and cc (m), respectively.

is a common problem in subsurface irrigation systems, and is usually solved by injecting herbicides through the pipes (Camp (1998)). In the CI system, this is hence avoided naturally by the high temperature of the pipe airflow.

Subsurface irrigation schemes in general are considered more water efficient since surface evaporation is reduced (Camp (1998)), harvesting becomes easier, and surface crusts are prevented (Lindblom (2006)). CI has the additional advantage of being able to safely use saline or otherwise polluted water as a source for irrigation. When using solar thermal energy for the air humidification, the irrigation rate then also follows the irrigation need: during sunny days, more water is produced in the subsurface pipes than on cloudy or cold days.

3

Mass & Heat Transfer in the Vadose Zone

3.1 Water Storage in the Vadose Zone

The ground is a porous material consisting of soil particles and pores filled with gas and liquid to a varying degree. At a certain depth the ground water table is found, where water exists as a free water surface (Marshall *et al.* (1996)).

Between the ground surface and the ground water table is the vadose zone. Here, the pores are only partly water filled, and the soil in this zone is therefore said to be unsaturated. The water in the vadose zone forms thin films along particle surfaces, as wedges around contact points of particles or as isolated bodies in narrow pore passages. The part of the pores not containing water is filled with pore gas, which generally consists of air with an excess amount of CO₂ due to bacterial activity (Koorevaar *et al.* (1983)).

The storage of water in the vadose zone is mainly a result of attractive forces between the soil particles and water, called Matric forces. Matric forces act against gravity, evaporation, root uptake, etc. and bind water to the soil particles by either:

1. direct adhesion of water molecules to solid surfaces by van-der-Waal forces. These forces are powerful but have a short range, so only a very thin layer of water is adsorbed around the soil particles. The forces are strong enough to prevent this water from being extracted by plant roots.
2. capillary binding, resulting from the adhesive attraction of water to soil particles and cohesive forces between the water molecules.

A third force to retain water in the soil is a result of osmotic binding of water to

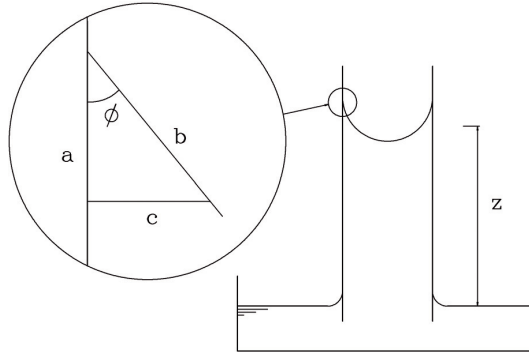


Figure 3.1: Water held in a capillary. After Koorevaar *et al.* (1983)

the soil due to solutes in the water. These forces are often ignored (Robinson and Ward (2000)) and lie outside the scope of present study.

3.1.1 Capillarity

Capillarity is one of the most important mechanisms in water storage and movement in the ground and is a result of water's adhesive attraction to the soil particles and cohesive forces between the water molecules.

The adhesion of water to a solid particle, γ (J m^{-2}), is characterized by the potential energy of the water molecules close to a surface. If the adhesive force is positive, the energy level in the molecules close to the solid is higher than that of bulk water, and the solid is said to be hydrophobic. Conversely, if the adhesive force is less than in the free water (i.e. $\gamma < 0$), the surface will attract water and the solid is called hydrophilic (Koorevaar *et al.* (1983)).

In the absence of adhesive forces, the cohesive forces between the water molecules strive to minimize the surface energy, and forms spherical drops of water, as can be seen when watching rain drops fall through the air. Inside the water drop the cohesive forces are the same in every direction, so the net force is zero. On the surface of the water drop, the molecules are only attracted inwards, resulting in a net force greater than zero. The surface tension, τ (J m^{-2}), describes this force.

Where water, air and solid surface meet inside a capillary, the shape of the water surface will be determined by γ and τ . The direction of the surface tension is always tangential to the meniscus (Fig. 3.1).

In Fig 3.1, a is the surface between the solid and water, b is the surface between air and water, c is a small arbitrary line segment normal to the solid surface, and ϕ is the contact angle between the water surface and the solid. The total contact

energy E (J) associated with the surfaces then becomes:

$$E = \gamma a + \tau b = \frac{\gamma c}{\tan \phi} + \frac{\tau c}{\sin \phi}.$$

The contact angle ϕ between the solid and the water is found where E reaches a minimum, i.e. $dE/d\phi \rightarrow 0$:

$$\frac{\partial E}{\partial \phi} = \frac{\tau c \cos \phi + \gamma c}{\sin^2 \phi} = 0,$$

resulting in

$$\cos \phi = -\frac{\tau}{\gamma}.$$

When $\phi = 0$, the water is said to completely wet the surface. In the hydrophilic capillary shown in Fig. 3.1, the water rises along the inside surface of the capillary until the capillary energy equals the potential energy of the water in the column. If the contact angle is assumed to be zero, then $-\tau = \gamma$, and the capillary energy can be calculated as the surface tension times the total contact surface between water and solid. The capillary energy can also be defined as the potential energy exerted by the water inside the capillary. Hence:

$$-2\pi r z \tau = -\rho_{w0} g z (\pi r^2 z)$$

in which z (m) is the capillary height, ρ_{w0} (kg m⁻³) is a reference water density, g is the gravitational constant, and r (m) is the radius of the capillary. The capillary energy is negative, since the pressure inside the capillary is negative. The capillary height of the water is hence a function of the inner radius of the glass tube (Marshall *et al.* (1996)):

$$z = \frac{2\tau}{\rho_{w0} g r}. \quad (3.1)$$

In non-cylindrical capillaries, such as soil pores, the equivalent radius is defined as the radius of a cylindrical capillary having the same height of capillary rise (Koorevaar *et al.* (1983)).

Pressure head, H (m), is defined by the energy required (J m⁻³) to lift one unit volume of water to the height H in the vertical direction, i.e:

$$H = \frac{p}{\rho_{w0} g} \quad (3.2)$$

Since the capillary height is defined by the potential energy in the capillary, it is per definition a pressure head, and can therefore be related to the capillary pressure, p_c (Pa), by:

$$p_c = \rho_w g z.$$

Because the capillary pressure is equivalent to the pressure difference between the water pressure below the meniscus and the air pressure above it, the following is also true:

$$p_c = p_w - p_a, \quad (3.3)$$

where p_a (Pa) and p_w (Pa) are the air and water pressures at the meniscus inside the capillary. The highest pressure is always on the concave side of the meniscus, so when there is a capillary rise, the capillary pressure will always be negative (Helmig (1997)).

3.1.2 The Soil Water Retention Curve

From Eq. (3.1), it is evident that the pore size distribution in the soil is of major importance for its ability to store and conduct water. In fact, the water content of a sandy soil is directly proportional to the capillary suction in the pores.

Generally, sandy soils have large pores while clay soils have small, so clayey soils retain water more strongly than sandy soils. However, even when a clay soil has a higher water content, it may still have less water available for plant uptake than a sandy soil, as a result of greater capillary suction. Figure 3.2 shows the relationship between water content and capillary suction of some typical sand and clay soils (Haverkamp *et al.* (1977); Brooks and Corey (1964); Jackson *et al.* (1965)). These curves are referred to as Soil Water Retention Curves (SWRC) and are characteristic for a given soil.

When describing water movement in an unsaturated porous medium, the SWRC is essential for determining the water content from a given capillary pressure or vice versa. The SWRC must hence be expressed in a functional form. In the current study, hysteresis effects are neglected, so the SWRC becomes a unique function for each soil type. The expression derived by Van Genuchten (1980) is used as default:

$$p_c = -\frac{1}{a} \left(S^{1/m} - 1 \right)^{1/n} \quad (3.4)$$

In Eq. 3.4 a , m and $n=1-1/m$ are shape parameters for the function and S is either the water saturation degree, S_w ($\text{m}^3 \text{ m}^{-3}$), or the effective water saturation,

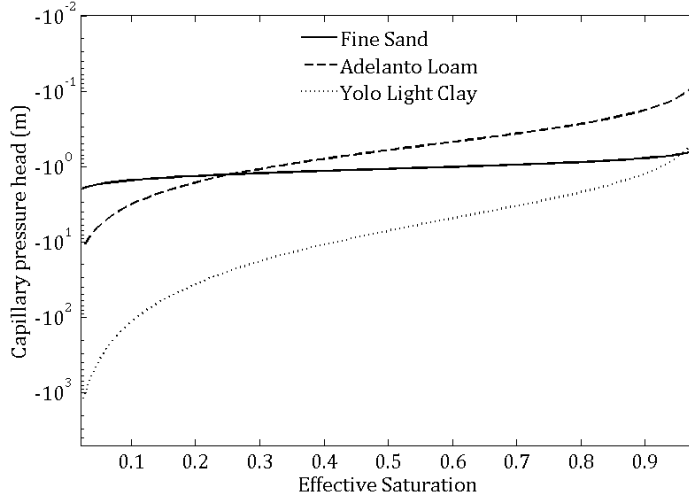


Figure 3.2: Soil Water Retention Curves for Fine sand (Brooks and Corey (1964)), Adelanto loam (Jackson *et al.* (1965)) and Yolo Light Clay (Haverkamp *et al.* (1977)), that are used in this study. The vertical axis shows the log scale of the capillary pressure head, H_c (m), and the horizontal axis the effective saturation degree, S_e .

S_e ($\text{m}^3 \text{m}^{-3}$). Because S_w is defined as the fraction of the total pore volume that is filled with water, then the gas saturation, S_g ($\text{m}^3 \text{m}^{-3}$) equals

$$S_g = 1 - S_w \quad (3.5)$$

In some cases, the soil is unable to drain completely, for example due to water trapped in isolated pores. This water amount, defined as the residual water saturation, S_r ($\text{m}^3 \text{m}^{-3}$), can therefore not be included in the water balance. In Eq. (3.4) the capillary pressure must then be calculated as a function of the effective water saturation, defined as:

$$S_e = \frac{S_w - S_r}{1 - S_r}$$

The expression by van Genuchten (1980) is a continuous function, which is a great advantage from a numerical point of view. However, as the water saturation approaches zero for high capillary suction a small change in the water saturation will cause an almost infinite change in capillary suction, which may cause oscillations in the numerical solutions.

3.2 Non-isothermal Flow in Unsaturated Soil

3.2.1 Fluid transport in the soil

Darcy's Law

When pressure differences arise in the soil, the inherent soil fluids will be move to balance the pressure gradients. The flow rate of the fluids will depend on the magnitude of the pressure gradient (i.e. the pressure difference per meter) and the ease with which the fluids can move through the pores in the soil. In a water saturated soil the only fluid present is water. Fluid flow is then described by Darcy's Law:

$$\nu = -k \frac{(p_2 - p_1)}{l_{12}} \quad (3.6)$$

in which p_2 (Pa) and p_1 (Pa) are the water pressures at some locations 1 and 2, l_{12} is the distance between these locations, and k ($\text{m}^2 \text{Pa}^{-1} \text{s}^{-1}$) is a proportionality constant called the saturated hydraulic conductivity. The negative sign indicates that the flow direction is in the opposite direction of the pressure gradient. The Darcy velocity, ν (m s^{-1}), is despite the unit, only an apparent velocity. The correct interpretation is actually flow rate per cross-sectional unit area. The Darcy velocity should therefore more accurately be termed flux density. The Darcy law is valid for flows with Reynolds numbers less than 1, which holds for fluid transport in soil.

It is possible to separate the Darcy proportionality constant k into two parts: one that is solely dependent of the fluid properties, and one that only depends on the porous grid structure. The latter of these is called the intrinsic permeability, k_i (m^2), and can be said to be a function of the solid particle diameter and porosity. The fluid based part of the saturated hydraulic conductivity is the fluid viscosity, μ_w (Pa s). The saturated hydraulic conductivity is hence defined as:

$$k = \frac{k_i}{\mu_w} \quad (3.7)$$

When including the intrinsic permeability in the Darcy law, and separating the total pressure gradient into an external pressure p_w (Pa) and potential (gravitational) pressure, the Darcy law becomes Helmig (1997):

$$\mathbf{v} = -\frac{k_i}{\mu_w} (\nabla p_w + \rho_w g \nabla y) \quad (3.8)$$

in which y is the vertical direction. The gradient of y is 1 in the vertical direction, and 0 in the horizontal, since the pressure head increases with 1 meter

per meter above the ground water table.

Flux density in the vadose zone

The Darcy equation, Eq. (3.8), is only valid for water transport in saturated porous media because the saturated hydraulic conductivity is based on a flux that has the whole pore space at its disposal. When the soil pores are only partly filled with water, water flux is reduced in several ways: firstly, the water flow area is reduced. Secondly, if the gas phase in front of the water flux is trapped in the pore channels, the gas pressure will build up and impede the water flux. Also, as the soil is dried, the remaining water is more and more tightly bound to the soil particles through the adhesive forces acting between the water molecules and the soil particles. This retentiveness is in other words the capillary suction, and is described by the SWRC.

This reduction of water conductivity in the vadose zone is accounted for by including the relative permeability, k_{rw} , which equals 1 when the soil is saturated, and decreases to 0 as the soil is dried to the residual saturation.

When the pores are drained of water, they become increasingly filled with pore gas. As the gas flow area increases, and water barriers disappear, the gas flux density increases. This relation is described by the relative permeability for the gas phase, k_{rg} , which increases from 0 to 1 as the water content decreases from fully saturated to dry soil. The variables k_{rw} and k_{rg} are here described using the theory by Van Genuchten (1980) in conjunction with Mualem (1976):

$$\begin{aligned} k_{rw} &= S_e^{1/2} \cdot \left(1 - \left(1 - S_e^{1/n}\right)^n\right)^2 \\ k_{rg} &= (1 - S_e)^{1/3} \left(1 - S_e^{1/n}\right)^{2n} \end{aligned} \quad (3.9)$$

To fit multiphase flow in unsaturated soil, a generalized form of the Darcy relationship (Eq. 3.8) is written as:

$$\mathbf{v}_\alpha = k_i \lambda_\alpha (\nabla p_\alpha + g \rho_\alpha \nabla y) \quad \alpha = g, w, \quad (3.10)$$

where $\mathbf{v}_\alpha$ (m s^{-1}), ρ_α (kg m^{-3}), and p_α (Pa), is the flux density, density, and pressure of either the gas (g) or the liquid (w) phase. The term $\lambda_\alpha = \frac{k_{r\alpha}}{\mu_\alpha}$ is referred to as the mobility of phase α (Helmig (1997); Forsyth *et al.* (1995)).

Gas diffusion

The gas phase in the soil generally consists of humid air with an excess amount of CO_2 , but for simplicity, air in the soil is assumed to have the same composition

as the ambient. The humidity in the soil, however, is quite different from normal ambient conditions. This is due to the capillarity and the adhesive forces acting between the water molecules and soil particles that keep the vapour concentration in the pores very near saturation. In fact, the relative humidity seldom or never sinks below 98%: in very moist sand, saturated to 90% with water (capillary pressure ≈ -300 Pa) the relative humidity is about $\varphi = 99.9998\%$. By draining water from the sand so that only 1% of the pores are water filled (capillary pressure $\approx -100,000$ Pa, i.e. near vacuum), the relative humidity sinks to $\varphi = 99.9260\%$. It is therefore reasonable to approximate the pore gas humidity to $\varphi \approx 100\%$ (Class (2009); Pruess (1987)).

The gas pressure in the pores, p_g (Pa), is the sum of the partial pressures of dry air, p_a (Pa), and vapour, p_v (Pa), the partial densities of the gas components are derived from the ideal gas law, and the mass fraction of the air (a) or vapour (v) component β , χ_β , is calculated as

$$\chi_\beta = \frac{\rho_\beta}{\rho_g} \quad \beta = a, v,$$

which means that:

$$\chi_a = 1 - \chi_v. \quad (3.11)$$

Concentration gradients within the gas phase arise continuously due to gas pressure or temperature gradients, or as a result of condensation or evaporation. In the strife towards a steadier state, the air and vapour components within the gas drift in the opposite direction of the concentration gradient according to Fick's law of diffusion. Within the pore space, the diffusion constant of an air-vapour mixture, D_θ ($\text{m}^2 \text{s}^{-1}$), must be adjusted for the tortuous path of the pores with a factor τ , and with a factor of $\rho_g \theta S_g$, since only a part of the cross-section is available for gas transport. The diffusion of either dry air (a) or vapour (v), $\mathbf{i}_\beta$ ($\text{kg m}^{-2} \text{s}^{-1}$) is hence described by:

$$\mathbf{i}_\beta = -\rho_g D_\theta \tau \theta S_g \nabla \chi_\beta \quad \beta = a, v.$$

Since the corrected diffusivity is the same for all components, and because of Eq. (3.11), it also follows that

$$\mathbf{i}_a = -\mathbf{i}_v.$$

3.2.2 Heat transfer in the soil

Heat transfer is driven by temperature differences. Inside a porous media, heat is transferred primarily through water movements in both liquid and vapour form. Air movements as well as conduction (i.e. heat diffusion) also contribute to the transport of heat. Radiation heat transfer is neglected in the macro-scale analysis due to the complex surface structure and low temperature differences usually occurring inside a pore.

The heat flux, $\mathbf{q}$ (W m^{-2}), through a control volume of soil is thus a result of the water and gas movements through the pore system and thermal conduction:

$$\mathbf{q} = h_a (\rho_a \mathbf{v}_g + \mathbf{i}_a) + h_v (\rho_v \mathbf{v}_g + \mathbf{i}_v) + h_w \rho_w \mathbf{v}_w - k_h \nabla T. \quad (3.12)$$

In the above, h_a , h_v , and h_w (J kg^{-1}), are the enthalpies for air, vapour and water, and k_h (W m^{-2}) is a local overall thermal conductivity for the porous media. The air and liquid water enthalpies are calculated based on constant specific heat capacities, and the enthalpy of water vapour is obtained from the liquid water enthalpy by adding to it the energy required for evaporating water at the specific temperature and pressure, denoted latent heat of vaporization, L (J kg^{-1}) (van Wylen *et al.* (1994)):

$$h_v = h_w + L.$$

The latent heat of vaporization is a major contributor to the heat transfer in unsaturated porous media. At room temperature $L \approx 2.5 \text{ MJ kg}^{-1}$, which is roughly 600 times greater than the energy required to raise the temperature of same amount of water by 1°C . This energy amount is taken from the water upon evaporation and so stored in the vapour. When the vapour condenses, energy corresponding to latent heat of vaporization at that temperature is released. Based on tabulated data found in van Wylen *et al.* (1994), the latent heat of vaporization is here assumed to vary with temperature according to:

$$L = 2503766.7 - 2444.8 \cdot T.$$

The thermal conductivity, k_h , is assumed to be a function of the water saturation according to (Falta *et al.* (1992); Emmert *et al.* (1995)):

$$k_h = 1.5 \cdot S_w + 1.$$

3.2.3 Surface evaporation and plant transpiration

At the contact surface between a wet body and ambient air, vapour is continuously exchanged. As the ground surface, and consequently the air above it, is heated by the sun, the vapour carrying ability of the air increases. A net evaporation from the surface occurs. If the evaporation rate is not restricted by a reduced water supply from the soil, the Potential Evaporation rate (PE) is achieved. Usually, however, the bare soil evaporation is often less than PE .

If the ground is cultivated, the plants will enhance evaporation by the act of transpiration, and the combined surface and vegetation evaporation is then sometimes called evapotranspiration (ET) (Robinson and Ward (2000)). The water supply for the transpiration process is taken from the soil water in the root zone, from where it is transported to the roots by the negative water pressure at the root surfaces. The magnitude of this negative pressure varies according to crop and soil depth.

Potential evapotranspiration

The potential evapotranspiration, PET (mm month⁻¹), can easily be estimated by the Thornthwaite model, since it only takes the average temperature and day length into account (Kumar *et al.* (1987)):

$$PET = 16 \left(\frac{dl}{12} \right) \left(\frac{10T_a}{I} \right)^a \quad (3.13)$$

in which dl (hours) is the average day length of the period considered, T_a (°C) is the mean ambient temperature, I is a heat index, calculated as

$$I = \sum_{i=1}^{12} \left(\frac{T_a}{5} \right)^{1.514}$$

and the exponent a in Eq. (3.13) is

$$a = 6.75 \cdot 10^{-7} I^3 - 7.71 \cdot 10^{-5} I^2 + 1.792 \cdot 10^{-2} I + 0.49239.$$

Bare soil evaporation

The actual evaporation rate, EV (mm h⁻¹) is in this study based on the pressure potential difference of water in the soil surface and in the air (Lappala *et al.* (1987)), so that

$$EV = 3600 \cdot \frac{k_{rw} \rho_w k_i}{\mu_w} \cdot \Delta y \cdot (H_a - H_w) \quad (3.14)$$

in which Δy is the vertical distance between the two topmost nodes in the section. H_a (m), is the pressure potential of water in the ambient air, which can be calculated by the Kelvin-Laplace equation (Collin *et al.* (2002)):

$$H_a = \frac{R_v T_a}{g} \ln \varphi$$

In a condensation irrigation system, some of the vapour flux from the pipe perforations leaves through the ground surface, and contributes to the surface water loss as well.

Heat exchange at the soil surface

During the day, the ground surface is heated from below by the buried drainage pipes, and from above, by the sun, corresponding to a heat flux of G_{sun} (W m^{-2}). Some of this heat is re-radiated to the ambient according to:

$$G_{sky} = \sigma \cdot \left((\bar{T}_{sky} + 273.15)^4 - (T_{surface} + 273.15)^4 \right)$$

in which $\sigma = 5.67 \cdot 10^{-8}$ ($\text{W m}^{-2} \text{K}^{-4}$) is the Stefan-Boltzmann constant, $\bar{T}_{surface}$ ($^{\circ}\text{C}$) is the mean surface temperature at the moment in question, and T_{sky} ($^{\circ}\text{C}$) is the effective sky temperature. Some heat is carried away from the surface by the wind, q_{wind} (W m^{-2}), according to:

$$q_{wind} = h_{wind}(T_a - T_{surface}),$$

where h_{wind} ($\text{W m}^{-2} \text{K}^{-1}$) is calculated as (Palyvos (2008)):

$$h_{wind} = 5.8 + 3.95 \cdot V_{wind},$$

in which V_{wind} (m s^{-1}) is the wind speed.

Root water uptake

There are many models describing root water uptake by plants (e.g. Jarvis (1989); Feddes *et al.* (2001); Van Dam *et al.* (1997); Simunek and Hopmans (2009)). However, since the present study of CI does not include any specific crop data, the following method was adopted from Lappala *et al.* (1987) instead: The ability of the roots to extract water from the ground varies in the vertical direction with the root activity function, $r(y,t)$ (m^{-2}). The root extraction, Q_w (kg s^{-1}), is determined by:

$$Q_w = \rho_w v \frac{k_i k_{rw} \cdot \rho_w 0g}{\mu_w} r(y, t) (H_{root} - H_w)$$

in which H_{root} is the root suction head, set to an assumed permanent wilting point of -150 m, and v_i (m^3) is the volume. The total evapotranspiration rate from the surface is hence:

$$ET = EV + Q_w$$

In surface water balance calculations the actual evaporation rate should always be lower than PET . In the following theoretical analysis, ET is only used as long as it does not exceed PET . In Condensation Irrigation, the soil surface is kept relatively dry, even during irrigation, which leads to lower surface water content, and thus less water losses, than conventional irrigation systems. This means that PET very seldom is reached.

3.3 Conservation of Mass & Energy

3.3.1 Mass balance of air

The pore gas is here assumed to consist of air and saturated vapour. Air is furthermore only considered in the gas phase, and although some air may be dissolved in the water phase, this amount is neglected. The air component in the gas is transported through the porous media by diffusion within the gas and by gas phase advection. Any accumulation of air within a control volume is a result of air movement retardation over the control volume together with any external air injection to the control volume (source). Accelerating fluxes through the volume results in a reduced air mass inside the control volume. This is summarized in the air mass balance:

$$\frac{\partial}{\partial t} (S_g \theta \rho_a) + \nabla \bullet (\bar{i}_a + \rho_a \bar{v}_g) = f_a \quad (3.15)$$

in which the term f_a ($\text{kg s}^{-1} \text{m}^{-3}$) contains any sources or sinks within the control volume, and $S_g \theta \rho_a$ (kg m^{-3}) equals the mass of air per unit volume. The time derivative of the air mass is expanded to

$$\frac{\partial}{\partial t} (S_g \theta \rho_a) = S_g \theta \frac{\partial \rho_a}{\partial t} + S_g \rho_a \frac{\partial \theta}{\partial t} + \theta \rho_a \frac{\partial S_g}{\partial t}.$$

Making use of Eq. (3.5), and assuming the porosity to be constant in time, the above is reduced to:

$$\frac{\partial}{\partial t} (S_g \theta \rho_a) = \theta \left((1 - S_w) \frac{\partial \rho_a}{\partial t} - \rho_a \frac{\partial S_w}{\partial t} \right).$$

The air density time derivative can be divided into a pressure related and a temperature related derivative:

$$\frac{\partial \rho_a}{\partial t} = \frac{\partial \rho_a}{\partial p_a} \frac{\partial p_g}{\partial t} + \left(\frac{\partial \rho_a}{\partial T} - \frac{\partial \rho_a}{\partial p_a} \frac{\partial p_v}{\partial T} \right) \frac{\partial T}{\partial t}.$$

Inserting Eq. (3.2) and designating

$$\begin{aligned} a_H &= \theta (1 - S_w) \frac{\partial \rho_a}{\partial p_a} \rho_{w0} g \\ a_S &= -\theta \rho_a \\ a_T &= \theta (1 - S_w) \left(\frac{\partial \rho_a}{\partial T} - \frac{\partial \rho_a}{\partial p_a} \frac{\partial p_v}{\partial T} \right) \end{aligned}$$

results in a final expression for the air mass balance (Lindblom *et al.* (2012)):

$$a_P \frac{\partial H_g}{\partial t} + a_S \frac{\partial S_w}{\partial t} + a_T \frac{\partial T}{\partial t} + \nabla \bullet (\bar{i}_a + \rho_a \bar{v}_g) = f_a. \quad (3.16)$$

3.3.2 Mass balance of water

In the vadose zone water movements occur in both gas and liquid phase. While liquid water mainly transports due to capillary forces (i.e. water saturation differences), the gas transport is strongly temperature dependent.

When the temperature in the soil rises, liquid water starts evaporating, thereby increasing the vapour pressure and density in the gas phase. As a result, vapour moves towards cooler regions, where it precipitates as liquid water. This allocation of liquid water from the warm to the cold region causes liquid water to be drawn from the cool area towards the heat source by capillary suction. This transport mechanism, known as the heat pipe effect, is often the dominant means by which heat is transferred in a porous media.

The water balance includes both the liquid and the vapour phase, so that any phase change of water does not influence the total mass balance of water. Evaporation or condensation will however alter the water saturation degree, thereby influencing the permeability, pressure, and temperature of all fluid phases. The water balance equation is hence:

$$\frac{\partial}{\partial t} (S_w \theta \rho_w) + \frac{\partial}{\partial t} (S_g \theta \rho_v) + \nabla \bullet (\bar{i}_v + \rho_v \bar{v}_g) + \nabla \bullet (\rho_w \bar{v}_w) = f_w,$$

in which, f_w ($\text{kg s}^{-1} \text{ m}^{-3}$) represents any water or vapour source or sink, $S_w \theta \rho_w$ (kg m^{-3}) and $S_g \theta \rho_v$ (kg m^{-3}) equals the mass of liquid and vapour per unit volume. Assuming constant porosity, a relative humidity of 100%, and that the water density is only a function of temperature, the time derivative is expanded to (Lindblom *et al.* (2012)):

$$v_S \frac{\partial S_w}{\partial t} + v_T \frac{\partial T}{\partial t} + \nabla \bullet (\bar{i}_v + \rho_v \bar{v}_g) + \nabla \bullet (\rho_w \bar{v}_w) = f_w \quad (3.17)$$

in which

$$\begin{aligned} v_S &= \theta (\rho_w - \rho_v) \\ v_T &= \theta \left(S_w \frac{\partial \rho_w}{\partial T} + (1 - S_w) \frac{\partial \rho_v}{\partial T} \right) \end{aligned}$$

The evaporation rate, E ($\text{kg s}^{-1} \text{ m}^{-3}$), inside a control volume can be derived from the increase of vapour in the gas phase that was not gained from either gas transport to the control volume or from a source:

$$E = \frac{\partial}{\partial t} (S_g \theta \rho_v) + \nabla \bullet (\bar{i}_v + \rho_v \bar{v}_g) - f_v \quad (3.18)$$

in which f_v ($\text{kg s}^{-1} \text{ m}^{-3}$) represents any vapour source or sink. From the expression it follows that water evaporates when $E > 0$ and condenses whenever $E < 0$.

3.3.3 Energy balance

The transfer of heat through the soil, $\mathbf{q}$ (W m^{-2}), occurs through heat conduction together with gas and liquid transport, but is greatly enhanced by the latent heat released or absorbed through water phase changes.

The energy balance for a control volume of porous media is expressed as:

$$\frac{\partial Q}{\partial t} + \nabla \bullet \mathbf{q} = f_e,$$

in which f_e (W m^{-3}) represents any heat source or sink, and Q (J m^{-3}) is the total enthalpy of the components comprising the control volume:

$$Q = \theta [h_w S_w \rho_w + (1 - S_w) (u_v \rho_v + u_a \rho_a)] + (1 - \theta) h_p \rho_p,$$

where u_a ($\text{J kg}^{-1} \text{ }^\circ\text{C}^{-1}$) and u_v ($\text{J kg}^{-1} \text{ }^\circ\text{C}^{-1}$) are the internal energies for air and vapour, respectively. The heat flux $\mathbf{q}$ (W m^{-2}) is calculated from Eq. (3.12)

(Lindblom *et al.* (2012)). Re-arranging,

$$e_P \frac{\partial H_g}{\partial t} + e_S \frac{\partial S_w}{\partial t} + e_T \frac{\partial T}{\partial t} + \nabla \bullet \mathbf{q} = f_e, \quad (3.19)$$

in which

$$\begin{aligned} e_H &= u_a a_H \\ e_S &= \theta (\rho_w h_w - \rho_v u_v - \rho_a u_a) \\ e_T &= \theta (1 - S_w) \left(\rho_v \frac{\partial u_v}{\partial T} + u_v \frac{\partial \rho_v}{\partial T} + \rho_a \frac{\partial u_a}{\partial T} + u_a \frac{\partial \rho_a}{\partial T} \right) \\ &\quad + \theta S_w \left(\rho_w \frac{\partial h_w}{\partial T} + h_w \frac{\partial \rho_w}{\partial T} \right) + (1 - \theta) \rho_p \frac{\partial h_p}{\partial T} \end{aligned}$$

The evaporative cooling that occurs when water evaporates can be determined by multiplying the evaporation rate E (Eq. 3.18) by the negative latent heat of vaporization, L .

3.4 The CI2D model

The presented theory was used to build a finite element simulation model of mass and heat transfer in unsaturated soils, called CI2D. The assumptions made within the model, and its validation and verification are published in paper III. Because of the very strong non-linearities inherent in the balance equations (3.16-3.19), they were modified to increase robustness of the simulation model. A new approach to the thermal two-phase flow theory was developed, based on the theories of Celia *et al.* (1990), Celia and Binning (1992), and Binning (1994). This so-called modified Picard method and details on the discretization of the balance equations, are found in the Appendix, together with information about the upwind technique used on the phase mobilities.

4

Research Findings

4.1 Simulations of Condensation Irrigation

The validated CI2D model presented in paper III was adapted to simulate condensation irrigation by combining the two-dimensional soil model with a pipe going through several two-dimensional sections in the normal direction. This modified model was used in paper IV to simulate a specific case of condensation irrigation, in a climate similar to Malta in early summer.

In the adaptation process the following additional assumptions and simplifications were made to the model:

Drainage pipe

- The inlet airflow properties to one irrigation pipe were constant during the daily irrigation
- The airflow inside the pipe was considered frictionless and the humid air was assumed to behave as an ideal gas
- Condensed water was assumed to spread uniformly through the pipe perforations in each soil section and thus distribute evenly around the pipe perimeter
- Liquid water was unable to re-enter the pipe

Ground surface

- No rainfall was included in the model
- The wind speed was assumed to be constant

- 50% of the ground surface was assumed to be shaded from incoming and outgoing radiation by the plant canopy
- The water pressure potential in the ambient air assumed a constant value of -1000 m, because the estimated risk of simulation instability exceeded the gain in accuracy by each time step calculating a new actual pressure potential from an assumed relative humidity (Lappala *et al.* (1987)).

4.1.1 Subsurface airflow through drainage pipe

As the warm and humid airflow enters the cooler underground irrigation pipes, vapour precipitates and forms liquid distilled water. This water and some humid air infiltrates the surrounding soil through the pipe perforations, thereby decreasing the mass flux through the pipes. Applying a control volume around the pipe, between two locations $i-\frac{1}{2}$ and $i+\frac{1}{2}$ on either side of soil section i , the airflow heat loss in a horizontal pipe, $q_{p,i}$ (W) is (Alvarez (1990)):

$$q_{p,i} = \dot{m}_{l,i-\frac{1}{2}} \left(h_l + \frac{c^2}{2} \right)_{i-\frac{1}{2}} - \dot{m}_{l,i+\frac{1}{2}} \left(h_l + \frac{c^2}{2} \right)_{i+\frac{1}{2}}$$

where $\dot{m}_l$ (kg s⁻¹), h_l (J kg⁻¹), and c (m s⁻¹) are the mass flux, enthalpy, and velocity of the humid airflow. The heat transferred from the pipe wall to the soil in the two-dimensional section i , $q_{p,i}$ (W), also equals the energy change of the airflow, i.e. the sum of convection heat transfer due to temperature differences and the rate at which heat is lost with the mass transfer through the pipe perforations to the soil in section i (Lindblom and Nordell (2012)):

$$q_{p,i} = A_p (h_D (T_l - T_p) + L h_m (\rho_l - \rho_p))_i + \dot{m}_{s,i} h_{l,i}$$

where A_p (m²), T_p (°C), and ρ_p (kg m⁻³) are the pipe surface area, wall temperature and vapour density, T_l (°C) and ρ_l (kg m⁻³) is the mean air temperature and density in the cross-section, and $\dot{m}_{s,i}$ (kg s⁻¹) is the mass flux through the perforations. The convection coefficient, h_D (W m⁻² °C⁻¹), is defined as:

$$h_D = \frac{Nu_D \cdot k_l}{D_p}$$

in which k_l (W m °C⁻¹) and D_p (m) are the thermal conductivity of air and the pipe diameter. The Nusselt number, Nu_D , for turbulent pipe flow was estimated from the correlation (Bergman *et al.* (2011))

$$Nu_D = \frac{\frac{f}{8} \cdot (Re_D - 1000) \cdot 0.71}{(1 + 12.7 \cdot (f/8)^{0.5}) \cdot (0.71^{2/3} - 1)},$$

in which Re_D is the Reynolds number, and f is the friction factor, given as

$$f = (0.79 \cdot \ln(Re_D) - 1.64)^{-2}.$$

If the airflow is laminar, then

$$Nu_D = 3.66$$

is used instead.

In moving from location $i-\frac{1}{2}$ to $i+\frac{1}{2}$, the mass flux decreases by the amount of vapour condensing in the control volume and by the mass of humid air leaving through the perforations, $\dot{m}_{s,i}$ (kg s^{-1}), in soil section i :

$$\dot{m}_{l,i+\frac{1}{2}} = \dot{m}_{l,i-\frac{1}{2}} - A_p h_{m,i} (\rho_{l,i} - \rho_{p,i}) - \dot{m}_{s,i},$$

in which A_p (m^2) is the pipe surface area. The mass convection coefficient, h_m (m s^{-1}), is derived from the Reynolds analogy (Bergman *et al.* (2011)):

$$h_m = \frac{h_D D_{av}}{\rho_l^{1/3} c_p^{1/3} k_l^{-2/3}},$$

and $\dot{m}_{s,i}$ is assumed proportional to the difference in the static pressure of the humid air in the pipe, $H_{l,i}$ (m), and the gas pressure in the soil outside the pipe, $H_{g,i}$ (m), at that section, according to:

$$\dot{m}_{s,i} = -A_h k_i \rho_{l,i} g \rho_{w0} \sum_{j=1}^n \frac{k_{rg,j}}{\mu_{g,j}} (\nabla H_{g,jk} + \nabla y)$$

where A_h (m^2) is the total perforated area of the pipe in the section. $\nabla H_{\alpha,jk}$ is the pressure gradient between the air flow static pressure in section i , node k , and the adjacent pipe wall nodes j in the same section i .

4.1.2 Initial and boundary conditions

In the case simulated in paper IV, the solar radiation data, G_{sol} (W m^{-2}), corresponded to Malta (35°N) in May, using a cloudiness index of 0.6 and a surface albedo of 0.3 (Table 4.1).

Table 4.1: Hourly mean solar irradiation, G_{sol} (W m^{-2}), used in the example simulations of a condensation irrigation system in Malta.

6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00
147	249	247	340	414	467	495	495
14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00
468	416	342	251	150	45	0	0
22.00	23.00	24.00	1.00	2.00	3.00	4.00	5.00
0	0	0	0	0	0	0	42

Above ground, the ambient air temperature, T_a ($^{\circ}\text{C}$), was assumed to vary according to:

$$T_a = 8 \cdot \sin\left(\frac{2\pi t}{3600 \cdot 24}\right) + 20$$

in which t (s) is the time. Other relevant parameters used in the example discussed in paper IV are presented in Table 4.2:

Table 4.2: Used values for relevant parameters in the example simulations

Wind speed	V_{wind}	1.7	m s^{-1}
Sky temperature	T_{sky}	4.8	$^{\circ}\text{C}$
van Genuchten parameter	a	0.96	Pa
van Genuchten parameter	n	6.9	
intrinsic permeability	k_i	$1.745 \cdot 10^{-12}$	m^2
Porosity	θ	0.377	$\text{m}^3 \text{m}^{-3}$
Temperature	T_l	70	$^{\circ}\text{C}$
Mass flux	$\dot{m}_l$	0.5	kg s^{-1}
Pressure	P_l	106330	Pa
Pipe diameter	D_p	0.2	m
Pipe depth	d	0.4	m
Pipe spacing	cc	1.2	m

4.1.3 Irrigation capacity of Condensation Irrigation

The reference CI system was simulated over a period of 30 days, although the diurnal temperature and saturation variations became steady after less than ten. Over the simulated time, the average water production rate due to pipe condensation was $2.59 \text{ kg m}^{-1} \text{ d}^{-1}$. The vapour flux through the pipe perforations contributed with an additional $1.55 \text{ kg m}^{-1} \text{ d}^{-1}$, so that the resulting irrigation yield using a pipe spacing of $cc = 1.2 \text{ m}$ became $3.44 \text{ mm}^{-1} \text{ d}^{-1}$ (Lindblom and Nordell (2012)). Because the airflow was cooled by the ground during the day, the condensation

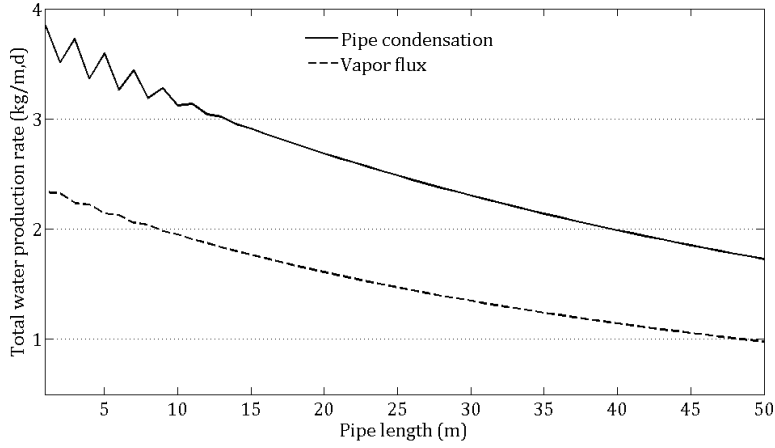


Figure 4.1: Calculated condensation rate along the first 50 m of a drainage pipe

rate declined along the pipe (Fig. 4.1), from a total of 5.1 mm d^{-1} at the pipe entrance down to 2.2 mm d^{-1} after 50 m along the pipe. This implies that short irrigation pipes are preferable to obtain a high irrigation yield.

During the first 30 days of irrigation, 6200 kg of water was released from one 50 m long irrigation pipe, of which 3880 kg was liquid water, and 2320 kg was vapour. Of this, 4% was lost through deep percolation, 19.8% was taken up by plant roots, 47.3% was accumulated in the ground control volume, and 28.9% was lost through the surface. The resulting mass balance of water over all 51 soil sections and 30 days was then $\varepsilon_w = -2.4 \cdot 10^{-4}$.

Of the surface loss, 688 kg was vapour, and 1104 kg was liquid water evaporation. This means that 70% of the vapour entering the soil with the humid air flux, formed freshwater in the soil and contributed to irrigation.

The amount of dry air injected through the pipe perforations was 31990 kg, of which 100.02% exited through the soil surface. 6 kg of the original air content in the soil was evacuated from the ground due to the increasing water content of the soil. The mass balance for air was hence $\varepsilon_a = -1.7 \cdot 10^{-5}$.

Over this period, the gas flux from the pipe had transferred $24.5 \cdot 10^9 \text{ J}$ of heat to the soil during the day, and removed $11.1 \cdot 10^9 \text{ J}$ during night air cooling, leaving $13.4 \cdot 10^9 \text{ J}$ to be distributed in the soil. Of this, 66.2% was released to the atmosphere, 13.7% was transported below the control volume, and 1.2% was used by the plant roots. The ground heating amounted to 18.6%, resulting in an energy balance of $\varepsilon_e = 2.8 \cdot 10^{-3}$.

Over the simulated period, the mean surface evaporation rate was 0.61 mm d^{-1} , and the mean vapour flux 0.38 mm d^{-1} . When adding the mean daily root transpiration of 0.68 mm d^{-1} , the average evapotranspiration rate becomes 1.67 mm d^{-1} .

When designing a CI system, the pipe configuration and inlet airflow parameters should be adjusted to the soil and climate so that irrigation needs are met and the temperature in the root zone is not too high. Variations of pipe configuration and inlet air parameters were therefore varied in the reference case to provide an example on how these influence soil temperature and irrigation yield (Fig. 4.2).

From Figure 4.2 it is evident that the airflow temperature has the highest effect on the irrigation rate of the system. It also appears that increasing the inlet air pressure and reducing pipe depth will yield more water. But even though the irrigation rate to the soil is increased, the increased gas flux to the surface in both these variations increases surface vapour loss even more, leading to less water in the ground.

An increased mass flux through the pipes increases irrigation yield, mainly through increased pipe condensation. Larger pipe diameter, using the reference inlet mass flux and pressure, increases the contact surface between the airflow and the soil, which enhances vapour condensation on the inner pipe wall.

By placing the pipes wider apart, the water production rate per pipe is increased, and the temperature in the soil is reduced. However, when crops are placed in the middle between two pipes, the wider spacing also increases the distance between the plant and the pipe, which makes it harder for plant roots to extract the supplied water. In the reference case, 19.8% (1228 kg) of the water supplied by the pipes was taken up by plant roots. When using a spacing of 2.0 m, root water uptake was only 0.8% (52 kg) of the supplied irrigation.

To obtain a higher root water uptake using a wide pipe spacing, crops should hence be placed closer to the pipe, at a distance still cool enough for the crop. Two rows of crop would then extract water from one pipe (Fig. 4.3), which means that the irrigation supply must be at least twice as large, and the soil temperature equal to or lower than the reference, in order to make this option more viable.

The double crop row option was simulated in paper IV with a pipe spacing and depth of 1.6 m and 0.4 m, respectively, and the inlet air temperature was maintained at 70°C . The plant roots were placed in a vertical line at a distance of 0.5 m from the pipe center (Fig. 4.3). The saturation and temperature of the mid-section of the pipe is shown in Fig. 4.4 after 30 days of irrigation at 18.00.

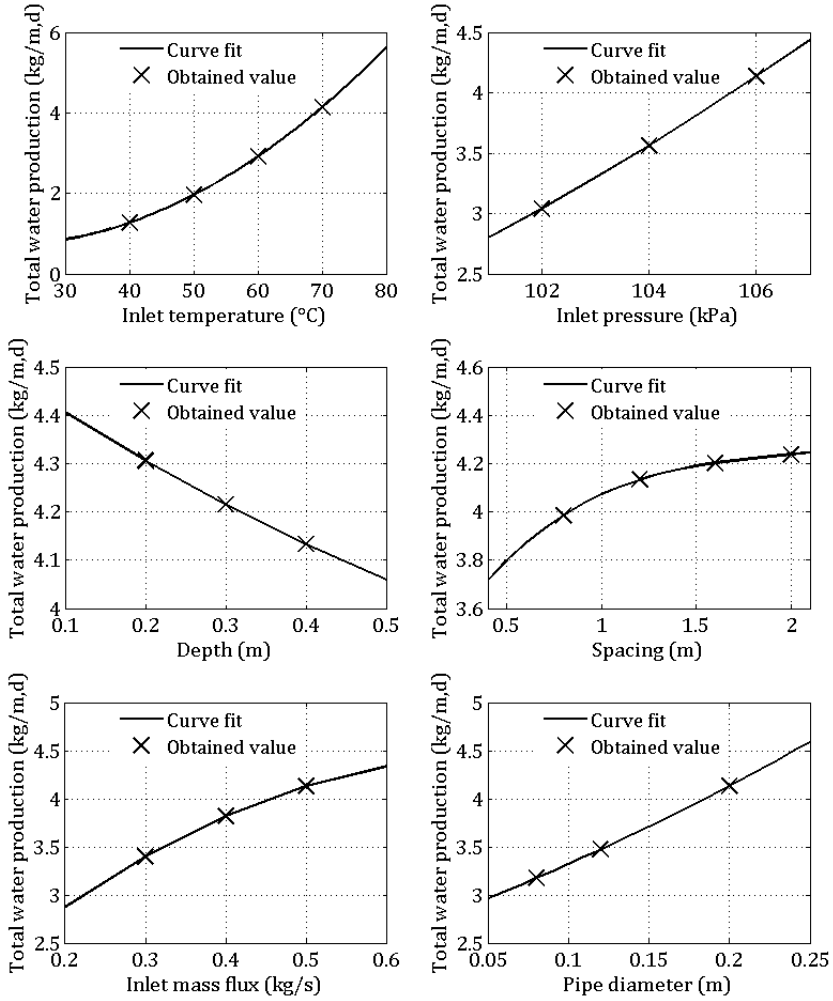


Figure 4.2: Influence of selected parameters on the irrigation rate in the simulated reference case

The obtained irrigation rate using double crop rows between the pipes was 2.64 mm d^{-1} ($4.22 \text{ kg m}^{-1} \text{ d}^{-1}$), and the root water uptake increased to 2938 kg, which is 2.4 times that of the reference scenario. The temperature in the rooting zone was also kept below 32°C . In the reference case, 19.8% of the water from the

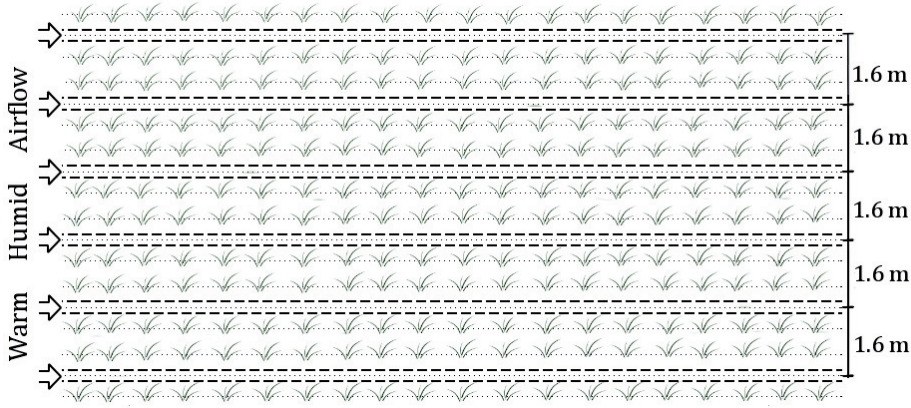


Figure 4.3: Alternative layout of the underground drainage pipes. Two rows of crops is planted between each drainage pipe.

pipes was used by the plants, while in the double crop row layout, the two rows of crop extracted 23.2% each of the supplied water. The double crop row layout is hence somewhat more water efficient in terms of irrigation. Another advantage with this layout is that less pipes are required: in the reference case, 82 pipes per 100 m are required, while using double crop row, 62 pipes will cover that same area.

By placing the pipes closer together it is possible to alter the reference system so that a sufficient irrigation is achieved using lower inlet air temperatures. In paper IV, this was demonstrated by reducing the inlet airflow temperature to 50°C and reducing the spacing to 0.6 m. After 30 days, the temperature in the root zone was still considered acceptable, but the irrigation was reduced to 3.00 mm d⁻¹ (Fig. 4.5).

The root water uptake was, however, increased from 1227 kg in the reference to 1289 kg, which means that 48.0% of the water supplied was used for water transpiration. This layout was hence more than twice as water efficient as the reference scenario.

Hausherr and Ruess (1993) performed field experiments on a condensation irrigation system with tomato plants in Switzerland. During the 12 week growing season, the total water amount supplied to the plants was 300 mm (3.6 mm d⁻¹), which is half the usual water requirement for tomatoes. In experiments performed by Zotarelli *et al.* (2009) on field-grown tomatoes in Florida a subsurface drip irrigation system was used. The water consumption in this system ranged from 2.6 to 4.6 mm d⁻¹ over the three year long survey. In relation to these irrigation

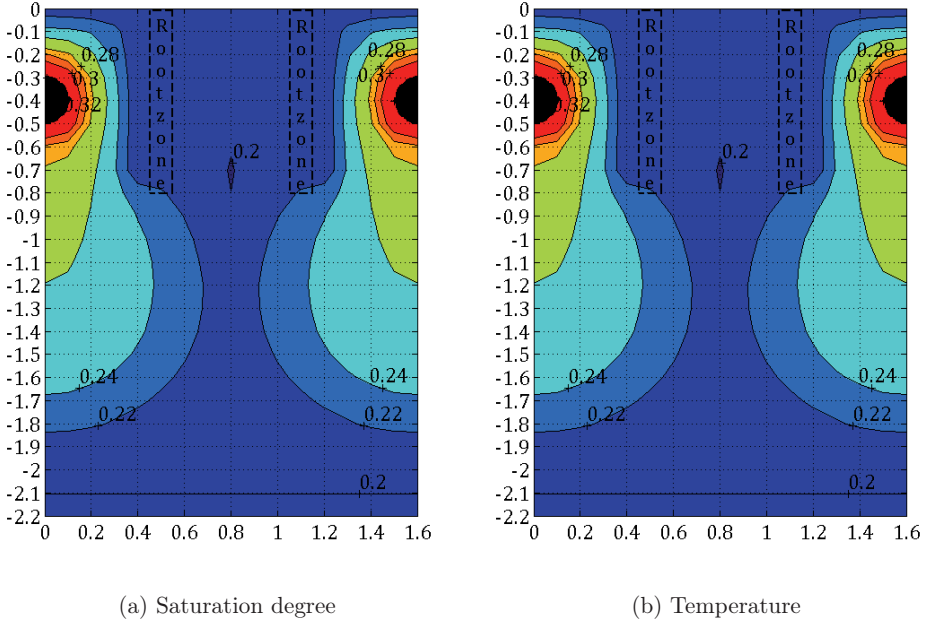


Figure 4.4: Soil water saturation, S_w , and temperature, T , in a soil with double crop rows. Sections at 25 m distance from the pipe inlet are displayed

rates the estimated water yield in a CI system is enough for field irrigation.

Regardless of airflow properties, the type of soil will ultimately decide whether condensation irrigation is a suitable irrigation system for a certain location. Because a large fraction of the heat and mass transfer occurs in the gas phase, the CI system will not work in soils with considerable water retention capacity. This is because the condensed water from the pipe then remains in the soil around the pipe, and cannot reach the plant roots. To illustrate this, the fine sand used in the reference was replaced by yolo light clay and by adelanto loam (Fig. 3.2 shows the SWRC of these). The water saturation 25 m along the pipe are shown for the three soil types in Figure 4.6.

In the simulated clay and loam, the total irrigation amount was smaller than in the reference sand, and the root water uptake was less than 1% in both the clay and loam. The condensation rate was, on the other hand, increased compared to the reference case, because of the very small quantity humid air leaking through the perforations. In the sand, the humid air flux is a source for irrigation water, but the gas flux and vapour condensation in the soil also increase the soil temperature, which reduces the condensation rate inside the pipes.

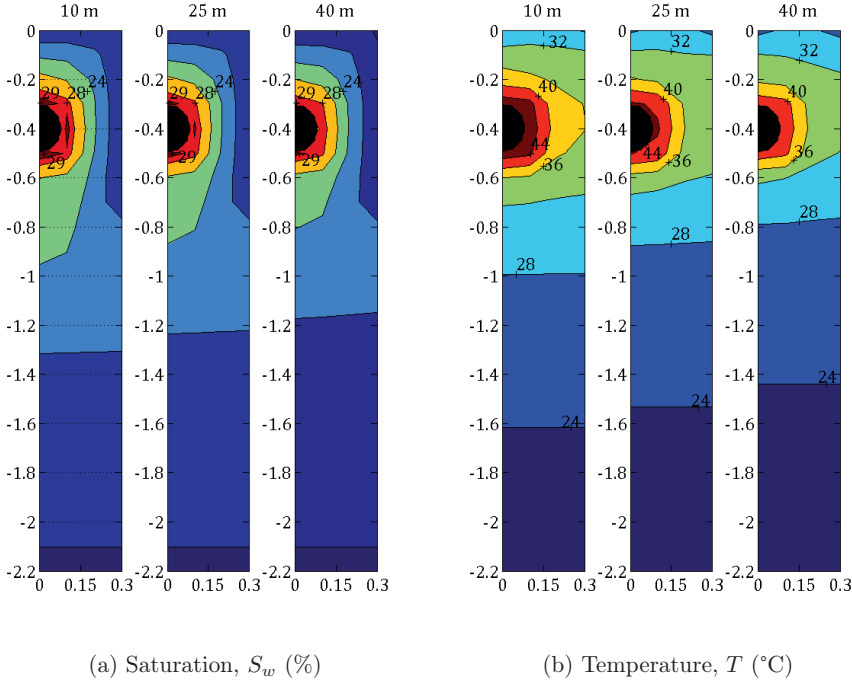


Figure 4.5: Soil water saturation and temperature using a pipe spacing of 0.6 m and an inlet air temperature of 50°C. Profiles are displayed for sections at 10, 25, and 40 m from the pipe inlet

4.1.4 Possibility for Drinking Water production

In a system for drinking water production, pipes without perforations are used in the ground. Since there is no vapour transport from the pipes in this case, the condensation rate inside such pipes increases. When replacing the perforated pipes with plain non-perforated pipes in the simulated reference case, the resulting water production rate was $2.7 \text{ kg m}^{-1} \text{ d}^{-1}$, i.e. 2.26 kg d^{-1} per unit land area (Lindblom and Nordell (2012)). Because the heat transfer in the soil is reduced in the absence of advective gas flux, pipes used for drinking water production could be placed closer together. It was found through simulations that the daily water production could be increased to 6.02 kg m^{-2} by using a pipe spacing of 0.4 m.

It is possible to design a drinking water system where no conventional solar still is used to heat and humidify the airflow. This could be achieved for example in coastal areas by leading in humid ambient air from the sea. To analyze this situation, pipes located at 0.4 m distance from each other, at 0.2 meters depth was

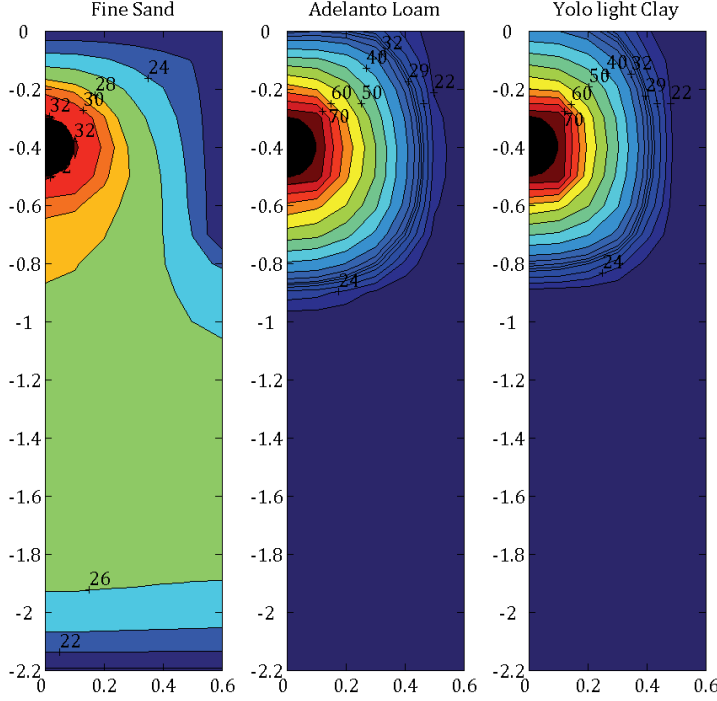


Figure 4.6: Water saturation, S_w (%) in section 25 m along the pipe, using the three different soil types described in Fig 3.2.

simulated with an inlet airflow temperature of 40°C. The daily water production rate along a 50 m long pipe was 0.51 kg m⁻¹. When using shorter pipes, the average water production rate increases: over the first ten meters, the mean water production rate was 1.72 kg m⁻¹, or 4.30 kg m⁻². Should temperatures of 50°C be obtainable in the above example, the mean water production rate in a 10 m long pipe would increase to 3.50 kg m⁻¹ (8.74 kg m⁻²).

4.1.5 Comparison of old and new models

The CI system was first investigated numerically in papers I and II, using a simplified simulator based on explicit FDM. It was then found that cooling warm humid air in a non-perforated pipe resulted in a daily water production of 1.8 kg m⁻¹ d⁻¹ (Lindblom and Nordell (2006)). Using perforated drainage pipes, resulted in a condensation rate of 3.1 kg m⁻¹ d⁻¹, but due to the simplified boundary

conditions, only 2.3 mm d^{-1} could be accounted for in the soil (Lindblom and Nordell (2007)).

Under similar circumstances as in paper II, the new CI2D model returns an irrigation yield of 2.4 mm d^{-1} . Water losses due to deep percolation and surface evapotranspiration are quite small; 0.3% and 9.2% respectively, so most of the water released from the pipes is accumulated in the soil. In the former model, the vertical water losses to the ground water and atmosphere were 38.3% and 23.4%, respectively.

The preliminary study on drinking water production by underground condensation of humid air in paper I, resulted in $1.8 \text{ kg m}^{-1} \text{ d}^{-1}$ of produced freshwater. This value was a slight overestimation since solar irradiation was neglected, but since no convective mass transfer was included in that model, the heat transfer in the previous model was also underestimated. By simulating the reference case used in paper I in CI2D, the water production rate was $1.8 \text{ kg m}^{-1} \text{ d}^{-1}$, i.e. the same value as in the 3D finite difference model in paper I. Since there is no gas flux from the pipe, the convective heat transfer in the soil is small in the system for drinking water. The simplification to only consider heat conduction in the soil, may thus be valid in this case.

4.2 Experiments on Condensation Irrigation

A small-scale CI plant was experimentally studied at LTU during 2010. The setup consisted of a thermally insulated sand box, connected to an air humidifier via a drainage pipe. The sand box was built 0.60 m high, 0.40 m across, and 0.20 m along the drainage pipe direction, and the pipe, located 0.225 m below the sand surface, was 50 mm in diameter. A thin cloth was wrapped around the pipe in the sand so that sand would not enter the pipe through the perforations (Lindblom and Nordell (2011)).

A 2.5 kW fan heater pumped ambient air into the humidifier, and another fan led warm and humid air from the humidifier into the drainage pipe going into the sand section. The humid air exiting the sand box was led back into the air humidifier, creating a semi-closed airflow loop. Inside the humidifier the air was humidified and further heated by steam rising from a water container heated by two electrical heaters.

Necessary sand properties were determined to enable theoretical simulations of the experiments. In Table 4.3, the measured porosity, particle density, and intrinsic permeability are shown together with two parameters used for creating the SWRC for the sand. The data used for the SWRC was obtained from a wetting experiment, which means that the calculated soil water content at a certain capillary suction could become lower than the true value in this sand. The intrinsic permeability was estimated from measuring the saturated Darcy flux through a sand column, Eq. (3.6), and applying Eq. (3.7). The obtained value is quite high, but could be explained by the small range in particle size used.

Table 4.3: Determined properties of the sand used in the CI experiment

Porosity	θ	0.4	$\text{m}^3 \text{m}^{-3}$
Particle density	ρ_p	2597	kg m^{-3}
van Genuchten parameter	a	14.857	m
van Genuchten parameter	n	5.1021	-
Intrinsic permeability	k_i	$8.38 \cdot 10^{-11}$	m^2

During the experiment, airflow properties were measured using Prandtl tube and a wet and dry thermocouple at the inlet and outlet of the sand box. The soil temperature was measured using eleven type T thermocouples placed vertically above and below the pipe, and horizontally at the height of the pipe center. Since the soil moisture content proved to be very difficult to measure during an ongoing experiment, sand samples were collected and weighed after completing a test run.

After drying in 105°C for three hours, the dry weight of the samples were used to determine the water content in each sample. The accumulated water in the soil profile, m_{sand} , was then estimated under the assumption that the soil water was evenly distributed around each sample.

From the difference in the measured wet and dry temperatures, the relative humidity could be estimated. By the air pressure inside the pipe, p_s (Pa), the vapour content, x (kg kg⁻¹) of the airflow was then calculated as:

$$x = \frac{0.622 \cdot p_{v,sat}^w}{p_{atm} + p_s - p_{v,sat}^w}. \quad (4.1)$$

The humid air velocity, v_g (m s⁻¹), was obtained from the dynamic pressure, i.e. the difference between the total, H_g^t (m), and the static pressure, H_g^s (m):

$$v_g = \sqrt{\frac{2 \cdot \rho_w g \cdot (H_g^t - H_g^s)}{\rho_g}},$$

where $\rho_g = \rho_a + \rho_v$ (kg m⁻³) is the humid air density. The mass flux of water vapour, $\dot{m}_v$ (kg s⁻¹), at the inlet and outlet of the drainage pipe could then be determined as:

$$\dot{m}_v = v_g \rho_g \psi A \cdot \frac{x}{1 + x}. \quad (4.2)$$

In the Eq. (4.2), a contraction coefficient, ψ , was multiplied to the pipe area because it was found that the flow was not fully developed at the inlet to the sand box. The mass flux of dry air, $\dot{m}_a$ (kg s⁻¹) could be determined by $\dot{m}_a = \frac{\dot{m}_v}{x}$.

The total amount of water leaving the pipe to the sand during one measurement, Δm_w (kg), was determined by the difference in vapour flux from the pipe inlet to the outlet of the sand column, times the time interval between two readings $\Delta m_w = \Delta t \cdot (\dot{m}_{v,in} - \dot{m}_{v,out})$. This water amount included both liquid water that condensed inside the pipe, and vapour flux through the perforations. The mass of dry air injected to the sand, Δm_a (kg), could also be obtained from the difference in mass flux at the in- and outlet to the sand box. Because the vapour content at these locations was known, the mass of vapour through the perforations, Δm_v (kg), could be determined as $\Delta m_v = \Delta m_a \cdot x$.

Because all of the sides but one in the sand box was airtight, all of the air leaving the pipe must exit the soil through the sand surface. Since this value was known, the vapour mass lost through the sand surface could also be estimated. By assuming that the air in the pores at the surface of the sand had the same

temperature as the bulk sand surface and that the relative humidity was $\varphi = 100\%$ (in a porous media $\varphi > 98\%$ even for very dry conditions), the vapour lost to the ambient between two measurement readings, $m_{surface}$ (kg), could be calculated as:

$$m_{surface} = \Delta m_a \frac{0.622 \cdot p_{v,sat}(T_{surface})}{p_{atm} - p_{v,sat}(T_{surface})}$$

An error estimate of the mass balance of water, ε , could now be determined as

$$\varepsilon = \frac{\Delta m_w - m_{surface} - m_{sand}}{\Delta m_w}$$

in which m_{sand} (kg) was the accumulation of water in the sand, estimated from the sand samples gathered in the sand column.

4.2.1 Main findings from experiments

In total eight experiment runs were carried out, comprising eight hours of irrigation followed by 16 hours of recess, with no airflow through the sand.

After three test runs it was concluded that the humid airflow was not fully developed at the inlet to the sand column, which led to errors in the mass flux calculations. The pipe length between the humidifier and the sand column was therefore lengthened before continuing with the five subsequent experiments. The longer pipe improved the flow profile of the air somewhat, but the fully developed flow was still not achieved. A longer pipe had additional effects on the flow, such as reduced flow rate, and cooler air entering the sand column.

When analyzing the water distribution based on the 24 sand samples collected after the three first runs, it turned out that the number of sand samples were too few. In the following five test runs, 66 samples were collected from the sand instead.

In the first run (Exp 1) the sand surface was covered with a plastic foil to seal off surface evaporation. The third run (Exp 3) was a repetition of the first experiment but without plastic foil. In the fourth and sixth runs (Exp 4 and Exp 6) irrigation was continued for three days, the fifth experiment (Exp 5) ran for 2 days, and the last two, Exp 7 and Exp 8, ran for 5 days each. The order of the experiments were selected randomly.

When comparing Exp1 and Exp3, the released water to the soil was found to increase from 0.69 mm d⁻¹ in Exp 1 to 1.31 mm d⁻¹ in Exp 3. Hence, by making the box airtight, the humid air flux from the pipe was prevented, which

greatly reduced the irrigation contribution from vapour condensation in the sand. Because the gas flux from the pipe greatly contributes to the heat transfer in the soil, the temperature was more efficiently removed from the pipe in the run without plastic cover (Exp 3). A warmer pipe wall in Exp 1 then also caused less vapour to condense inside the pipe, reducing this water source to the sand as well.

Neglecting the gas phase is a common assumption when simulating water flow in unsaturated porous media. This is however only a valid assumption as long as the gas has a free flow path to the surface. The effects of gas barriers on downward water infiltration have been clearly demonstrated in isothermal experiments performed by Touma and Vauclin (1986). The difference in water saturation and temperature distribution in Exp 1 and Exp 3 also shows the necessity of including the gas phase in models with temperature gradients. The gas phase then often becomes the dominant mode of heat transfer in the soil and can hence never be neglected.

The result of the reduced heat transfer in the sand is visible in the temperature profiles (Fig. 4.8), in which the temperature above the pipe is 40°C in the sand with surface cover (Exp 1) and only 34°C in the experiment without (Exp 3). The density of the isotherms indicates the heat transfer rate in different directions. In Exp 3 the isotherms are less dense upwards because the heat transfer is greatest in the upward direction. In the absence of gas advection to the surface, there is no such significant heat transfer direction in the temperature profile from Exp 1.

The moisture profiles in Fig. 4.7 show the reduced water content in the sand in Exp 1 compared to Exp 3. The soil water in Exp 1 was transported upwards, away from the warm pipe wall, while in Exp 3, the water accumulated closer to the pipe. In the area with the highest moisture in Exp 1, the sand temperature was found to be lower than the surrounding, since liquid water accumulates in the coolest region of a porous media. The reason for this upward motion of the water could be a result of leakage through the plastic cover at the edges, allowing some humid air at the sides to slip through. The gas pressure gradient would then have a sharp downward direction, preventing downward water movement.

4.2.2 Theoretical simulations of experiments

Three of the experiment runs were theoretically simulated using the computer program CI2D. The resulting mass balances of water from the experiments and simulations are presented in Table 4.4, together with the mass balance error.

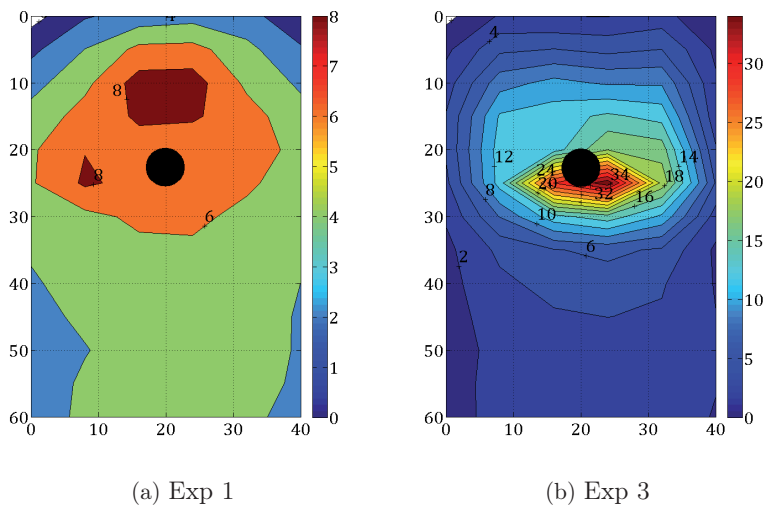


Figure 4.7: Saturation profiles (%) for Exp 1 and Exp 3.

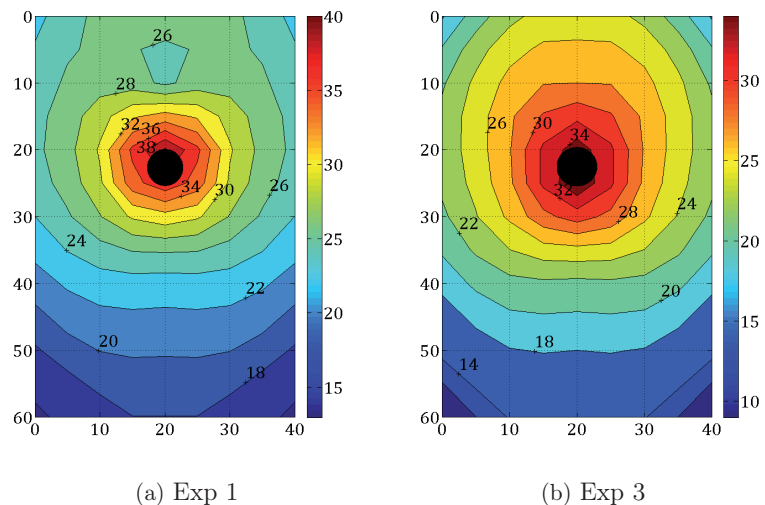


Figure 4.8: Estimated temperature profiles (°C) for Exp 1 and Exp 3

Table 4.4: Mass balances from experiment runs Exp 5, Exp 6, and Exp 8, and the corresponding values obtained through theoretical simulations.

Run	Experiment				Simulation			
	Δm (g)	$m_{surface}$ (g)	m_{sand} (g)	ε (%)	Δm (g)	$m_{surface}$ (g)	m_{sand} (g)	ε (%)
Exp 5	246	52	187	3%	244	52	192	0.08%
Exp 6	395	131	246	5%	397	150	248	0.2%
Exp 8	951	553	388	1%	852	463	393	0.5%

In the simulation of Exp 8, the water amount accumulated in the sand and the temperature was in good agreement with measured data. The estimations of the air flux in the pipe and sand was, however, much larger than the simulated values. This indicates that the contraction coefficient of the inlet airflow set in this run, $\varphi = 0.964$, was too high. The average value used in the other experiment run evaluations, $\varphi = 0.926$, would have estimated the airflow closer to the real value.

The measured and simulated temperature and saturation profiles in the soil are shown in Figs. 4.9, 4.10, and 4.11.

The soil temperature was in the experiments only measured to the right side of the drainage pipe. The thermocouples were there attached to a thin wooden stick, so they would remain at equal distances through all experiments. However, it turned out that the stick conducted water, which led to an uneven distribution of water to the right side of the pipe.

In the experiments, the freshwater formed inside the pipe percolated into the sand through the lower part of the pipe perimeter, which can be seen in Figs 4.9a, 4.10a, and 4.11a. The increased water saturation below the pipe diverted the humid air flux coming through the perforations to the upper part of the pipe, from where it transported to the surface. Since gas advection was the main heat transfer mechanism, the heat was more efficiently removed from the upper part of the pipe, causing the temperature below the pipe to be 1-2°C higher than above it (Figs. 4.9c, 4.10c, and 4.11c).

In the simulation model, however, the condensed water from the pipe was assumed to distribute evenly around the pipe perimeter. Initially, this assumption gave no preferential path for the gas flux. Nevertheless, as the water accumulated around the pipe, the water permeability increased and enabled a gravitational, downward movement in the sand, which gradually caused the soil to become wetter below the pipe (Figs 4.9b, 4.10b, and 4.11b).

Comparing the simulated and measured temperature profiles reveals that dif-

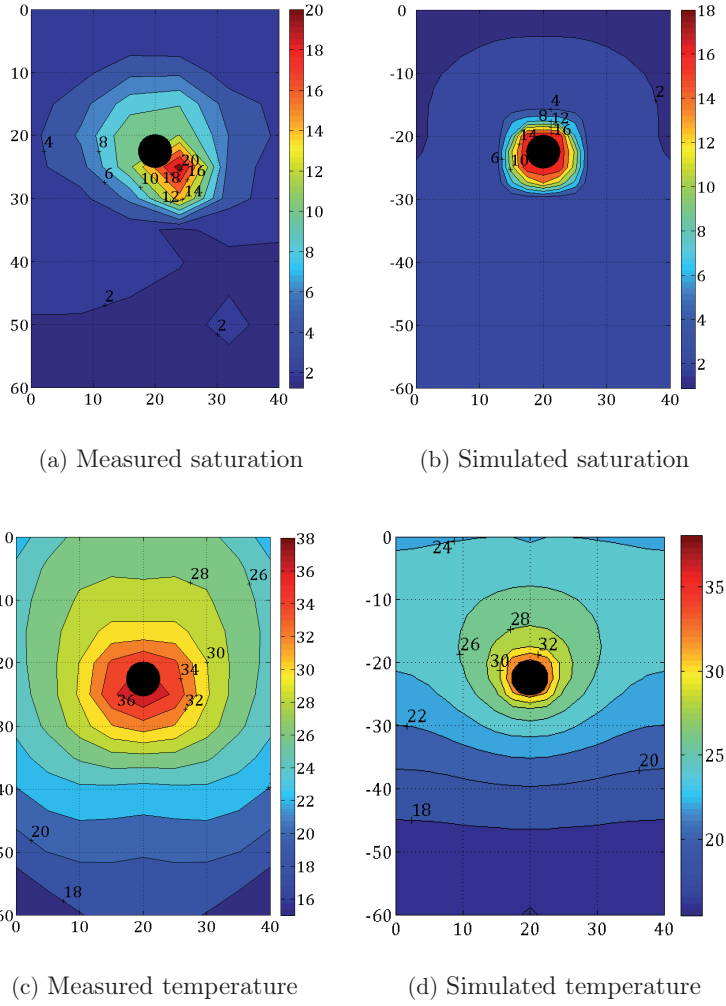


Figure 4.9: Measured and simulated profiles of soil saturation, S_w (%), and temperature, T ($^{\circ}\text{C}$), for Exp 5 (2 day run)

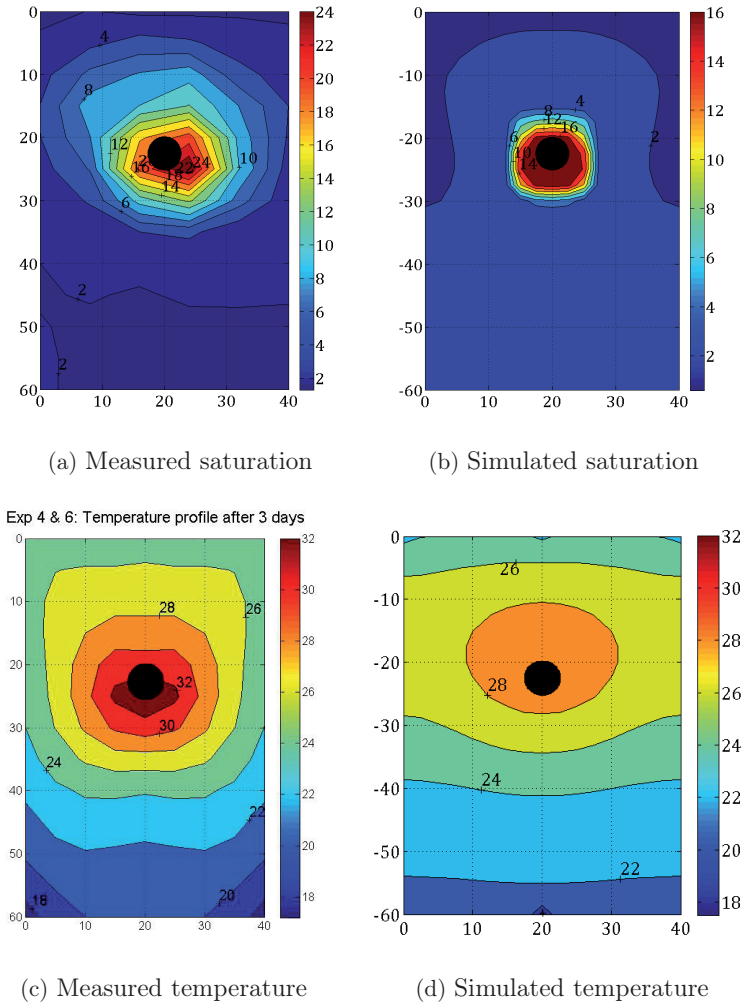


Figure 4.10: Measured and simulated profiles of soil saturation, S_w (%), and temperature, T ($^{\circ}\text{C}$), for Exp 6 (3 day run)

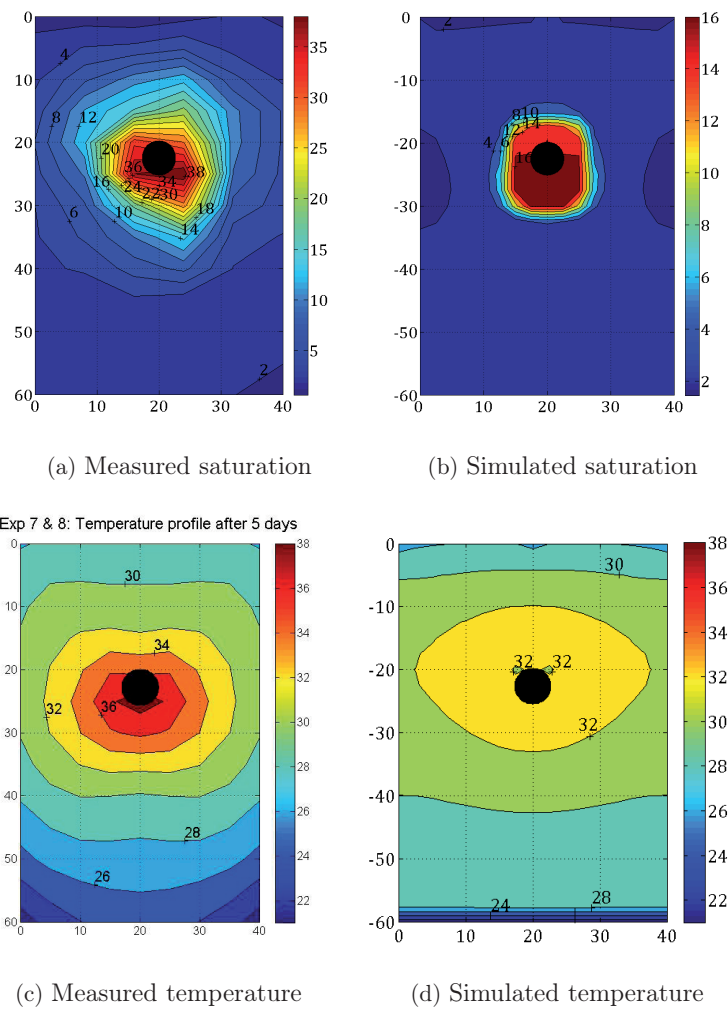


Figure 4.11: Measured and simulated profiles of soil saturation, S_w (%), and temperature, T (°C), for Exp 8 (5 day run)

ferent temperature levels are observed in the 2 day simulation. The water saturation around the pipe is also uniform around the pipe after the 2 day simulation. However, as the water concentrates more below the pipe, the simulated temperature levels become similar to the experimental values. There is a horizontal difference in the isotherms between the simulated and experimental profiles, resulting from imperfect thermal insulation in the experiments.

In all three comparisons, the saturation is more widely spread in the experiment profiles. One possible reason for this is that the assumed correlation for the relative gas permeability, Eq. (3.9), underestimated the true value for the sand used in these experiment simulations. There are many other available estimations of k_{rg} (e.g. Corey (1954); Fatt and Klikoff (1959); Pruess *et al.* (1999)), but to get the most accurate results, k_{rg} and k_{rw} should have been measured specifically for the used sand.

Another reason for the deviations in saturation profiles could be the lack of data required for the simulation model. In order to conduct numerical simulations of the system setup, hourly values for the ambient air temperature, pressure, and humidity is necessary. The initial sand temperature must also be known. Since these values were not recorded, they have been assumed in the simulations. Many of the insecurities in the results could have been abated by assuring a stable and uniform profile of the airflow inside the pipe. More repetitions of the same experiments would also have strengthened the relevance of the findings. By systematically varying the inlet airflow properties and the pipe diameter and depth, design parameters could more readily have been extracted from the results. Due to limitations in available time, this was not possible.

When increasing the duration of the experiments, the daily irrigation yield decreased from 1.17 mm d^{-1} over the first two days to 0.97 mm d^{-1} over five days operation. This reduction is a result of the gradually warmer sand around the pipe, which reduces the condensation rate in the pipe. As predicted, the reduction seemed to decline with time, so that the irrigation could reach a steady yield after a few days.

By cooling the sand at night, the daily irrigation yield could have been maintained from one day to another. In a field using CI, the cooling can for example be solved by circulating cool ambient air through the pipes at night, when no solar energy is available to power the irrigation.

Neither the experiments nor the simulations showed any water surface loss due to liquid water evaporation. The reason for this was the dry sand near the surface.

5

Conclusions

Condensation Irrigation (CI) is a combined system for desalination and irrigation. The desalination of saline water is designed as an air humidification-dehumidification process, in which ambient air is humidified inside solar stills. The warm and humid air is led from the stills into underground horizontal pipes, where it is cooled by the surrounding soil and vapour precipitates as freshwater.

If the pipes are perforated, the formed water percolates into the soil around the pipe as irrigation water. Humid air also leaks through the perforations and transports towards the ground surface. Some of the vapour contained in that air condenses in the soil and contributes to irrigation. In a pipe without perforations, the formed freshwater can be collected at the pipe endings and used for for example drinking. A small inclination towards the pipe outlet is then necessary.

A two-dimensional thermal two-phase simulation model was developed for obtaining theoretical predictions of the irrigation potential of a CI system. The final model, CI2D, was used for studying systems for both condensation irrigation and drinking water production, and for simulating the performed small-scale laboratory experiments.

5.1 Simulations of a Theoretical CI system

A theoretical condensation irrigation plant was simulated for climate conditions similar to Malta in May. Since no solar still was included in the simulations, the inlet airflow properties to the pipe were assumed and constant. The simulator CI2D produced stable solutions in all simulations accompanied by insignificant mass balance errors. In a pipe system with a spacing of 1.2 m between parallel pipes at 0.4 m depth, the obtained irrigation rate was 3.44 mm d⁻¹, although only 19.8% of this water was picked up by the plant roots. The temperature near the roots was then also about 32°C, which was considered the limit for crop tolerance.

To increase the relative crop water uptake, plants would need to be located closer to the pipe. This was not possible in the reference case, because the soil would be too warm. By placing the pipes wider apart, the heat released from one pipe was distributed in a larger soil volume, which increased the water production and lowered the temperature in the soil. One plant row could then be placed closer to a drainage pipe, and thus pick up more water. Since the crop row was not placed in the middle between two pipes, one pipe had to supply two crop rows with irrigation water. The crop water uptake in this double crop row outline rose to 46.4% of the supplied water.

The inlet airflow temperature was found to be the parameter with the highest influence on the irrigation rate, so by decreasing the inlet air temperature from 70°C to 50°C, the supplied irrigation dropped by 53%. However, when the pipe distance was also reduced by half, the root water uptake increased compared to the simulated reference case, and 48% of the supplied water was used for plant transpiration.

Simulations in soils with high capillarity, showed a significant reduction in water transport in the soil and root water uptake by plants. Condensation irrigation is hence not suitable in locations with clay soils and loam.

5.2 Studies on Drinking Water Production

By replacing the drainage pipes in the reference CI system with plain pipes, fresh-water could be collected at the pipe endings. Two scenarios of this system were simulated using CI2D. In the first, the same inlet parameters as the irrigation system were used. Due to reduced heat release to the ground, the vapour condensation inside the pipe was $2.7 \text{ kg m}^{-1} \text{ d}^{-1}$ along the 50 m long pipe. To improve the reference system, the pipes were placed at a distance of 0.4 m. The water production was then $2.4 \text{ kg m}^{-1} \text{ d}^{-1}$, which meant an increased water production per unit land area.

A second scenario of the water production system was studied, where cooler air with high humidity was taken from other sources than a solar still, such as warm air from the sea. Simulations using 40°C and 50°C humid air at the inlet, indicated a potential water yield of 1.75 and $3.50 \text{ kg m}^{-1} \text{ d}^{-1}$, respectively, over the first ten meters of pipe.

When comparing previous, simplified models of condensation irrigation and drinking water production, the irrigation and water production rates using the CI2D program were close to previous results, although the water and temperature distribution in the condensation irrigation application differed between the models.

5.3 Experiments of Underground Condensation of Humid Air

A small scale experiment was set up at LTU in which warm, humid air was blown through a perforated pipe buried in a sand box. The purpose of the experiment was to study heat and water transfer in the sand around the pipe.

By sealing off the sand surface with a plastic cover, gas advection in the sand was prevented, which resulted in much higher sand temperature around the pipe, and much less (53%) irrigation yield than in the reference experiment. This indicated that the gas advection was an important factor in the heat transfer. Gas advection should hence never be neglected or assumed constant in simulations of heat and mass transfer in porous media.

Without surface cover, the gas advection transported heat and humid air from the pipe wall to the ambient air above the sand surface. In doing so, the temperature around the pipe was greatly reduced, which increased the irrigation yield. Before constructing a CI plant, it is hence necessary to consider the porosity of the soil, so that gas advection is allowed in the soil.

Although missing important measurements, the experiments were simulated in CI2D, using assumed values where no data were available. The distribution of mass between water flux to the soil, water accumulated in the soil, and vapour flux through the surface could be recreated fairly well in the simulations, with the exception of the last performed experiment. The simulated temperature profile in the sand agreed with the experiment result, but the water distribution in the simulations was not as advanced as the profile found in the experiments. The probable cause was that the relative gas permeability correlation used in CI2D was not suitable for the sand used in the experiments.

5.4 Improvements of Current CI system

There are a number of improvements that could be added to the current outline of the CI system:

1. Because the daily airflow cooling and vapour condensation declines along the pipe (Fig. 4.1), short irrigation pipes are preferable to obtain a high mean irrigation yield in a pipe. By designing the CI system so that parallel pipes in the field have opposite flow directions, the irrigation supply along the pipes would become more evenly distributed. Using opposite flow directions in the pipes would also reduce the soil temperature, and hence increase total irrigation yield. Since this layout would involve two air humidifiers located

on the opposite sides of the field, the airflow from the first humidifier would exit the irrigation pipe into the second humidifier. This would also increase the air humidification efficiency, since air exiting a pipe already is vapour saturated and considerably warmer than the ambient air.

2. The saline inlet water the solar still usually originates from a relatively cool water source, such as the sea or the groundwater. By preheating the saline water before entering the still, the efficiency of the still is improved. This preheating could also be used to improve the irrigation in a CI system in one of two ways:
 - (a) By leading the saline water through a pipe running inside the perforated irrigation pipe, the cool water pipe would enhance the humid airflow condensation rate. To even out the irrigation yield along the irrigation pipe, the water flow direction should be the opposite of the airflow
 - (b) To ensure a low temperature in the root zone, the feed water pipe could be placed in the soil close to the roots. The root cooling caused by the saline water pipe would have the secondary effect of condensing vapour in the root zone, and thus increase the available water for roots. The flow direction of the water should then be the same as the humid airflow, so that the coolest water passes through the warmest soil.

5.4.1 Greenhouse climate control by CI

Hietala Market gardens in Övertorneå, Sweden, is a commercial greenhouse farm for growing cucumbers and other vegetables. During summer, the air temperature below the greenhouse ceiling can reach 50°C and is then also saturated with vapour. As some of this vapour in the afternoon condenses, droplets fall on the cucumber leaves, which may cause damage to the plants.

A climate control system was installed in 1987 in one of the cucumber greenhouses (Nordell (1987)). The system consisted of drainage pipes leading warm, humid air from the ceiling, through the ground among the cucumber roots, and up into the ambient air in the middle of the greenhouse. When circulating through the soil, the warm air heated and humidified the soil, but also increased the soil aeration, which was otherwise perceived as a problem in the compact compost soil. The air returning to the greenhouse was both cooler and drier, which reduced drop formation on the plant leaves.

By initializing the air circulation during spring time, the soil was heated by the solar heated air inside the greenhouse, which advanced the planting date of the cucumbers seeds.



Figure 5.1: Greenhouse at Hietala Market Gardens, Latitude 66.5°

The system was operational for nearly 20 years, but was finally dismantled in 2004.

Greenhouse climate control in a warm climate

In a warm, arid location, the greenhouse climate could be controlled by circulating the air below the ceiling through a pipe buried in the ground outside, and leading it back into the greenhouse. The condensed water inside the air circulation pipe could then be collected and used for drinking or irrigation, and the air returning to the greenhouse would be both cooler and drier.

From simulation results using the program CI2D, a 0.5 kg s^{-1} humid air flux at 50°C through a 50 m long buried pipe without perforations would generate 54 kg of freshwater per day. The temperature and humidity of the air returning to the greenhouse would thereby be reduced. Further studies on the greenhouse system for both cold and warm climates could disseminate the possibility for this application.

5.4.2 Recommendations

The CI system presented in this thesis has proven to be a technology with great potential with many conceivable applications. Due to the success of the climate control system at Hietala market gardens, the principles behind CI are thus applicable for greenhouse climate control and irrigation water re-use as well.

As a continuation of current work, it is recommended that the potential of the suggestions stated above should be investigated. To put theories to the test, a closely monitored pilot plant of a basic CI plant should also be built and studied over several crop seasons.

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Paper I

Water production by underground condensation of humid air

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Water production by underground condensation of humid air

Jenny Lindblom*, Bo Nordell

Renewable Energy Systems, Luleå University of Technology, SE-97187 Luleå, Sweden
Tel. +46 (920) 49 13 30; Fax +46 (920) 49 16 97; emails: Jenny.Lindblom@ltu.se, Bo.Nordell@ltu.se

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Abstract

Condensation irrigation (CI) is a combined system for desalination and irrigation. By evaporating seawater in, for example, solar stills and letting the humidified air transport the formed vapour into an underground pipe system, fresh water will precipitate as the air is cooled by the ground. By using drainage pipes for underground air transportation, perforations in the pipes enable the water to percolate into the soil. This study of CI focuses on the transport of humid air inside buried plain pipes, where the condensed water stays inside the pipe and may thus be collected at the pipe endings and used for drinking. Numerical simulations of this system result in a mean water production capacity of 1.8 kg/m and day over a 50-m long pipe in a diurnally steady system, though shorter pipes result in a higher mean production. A performed theoretical analysis also indicates that CI is a promising alternative irrigation method as it enables the use of saline water for irrigation.

Keywords: Desalination; Condensation; Irrigation; Drinking water; Humid air

1. Introduction

There is a severe lack of fresh water in the world today. Along with the deterioration of existing water supplies, the growing world population leads to the assumption that two-thirds of the population will lack sufficient fresh water by the year 2025 [1]. To meet the food demands of the estimated 7.9 billion people, cereal production must grow by a projected 47% between 1995 and 2025 [2].

Today's irrigated lands constitute 20% of the total cultivated area, contribute 40% of the total food production, and consume 72% of the annual world water withdrawal [2,3]. During the 20th century, irrigation in agriculture increased nearly fivefold. Without this increase in irrigation, crop yields would have been unable to meet the increased food demand of the current population level [2].

Although irrigation is vital for sustaining a high level of food production, several of today's methods are inefficient and contribute to land

*Corresponding author.

degradation by, e.g. salinization or waterlogging. Intensive irrigation in areas with poor drainage leads to a rise in the groundwater table, causing salinity build-up in semi-arid and arid regions and waterlogging in humid zones. Around 66% of global agricultural land has been degraded to various degrees over the past 50 years [2], of which 20–30% is affected by salinization or waterlogging [3].

The growing population necessitates further irrigation expansion of agricultural lands, though in many regions water for irrigation competes with non-agricultural sectors where economic gains from the use of water are higher [3]. The increasing cost of water entails new ways of producing freshwater.

Desalination of seawater is the only sustainable way to produce more freshwater. Each day, $23 \times 10^6 \text{ m}^3$ is produced by reverse osmosis, multi stage flash and multi effect distillation. The majority of this industry uses fossil fuels for the desalination process; only 0.02% is based on renewable energy according to Delyannis and Belessiotis, quoted in [4].

This paper outlines a combined system for solar desalination and irrigation/drinking water production, where the ground acts as a condenser for humid air flowing inside a system of buried pipes. In the irrigation system, the condensed freshwater is supplied directly in the root-zone through perforations in the pipe walls. This sub-irrigation scheme minimizes crop water consumption and eliminates water losses, such as surface runoff and evaporation. By moderating daily water distribution, the risk of land degradation is reduced, since salt transport to the topsoil by a rising groundwater table is prevented. The perforations also allow the irrigation system to be a drainage system that effectively removes excess soil water from the field. This analytical study is, however, mainly concerned with condensation of humid air in pipes without perforations. The condensed water may instead be collected at the pipe endings and used for drinking.

Previous studies of condensation irrigation (CI) also exist. Widegren (1986) performed theoretical studies on a CI system that irrigated a land area of 1 ha by using a fan power of 3–10 kW [5]. Nordell (1987) constructed a small-scale plant in a greenhouse for cucumbers in Övertorneå, northern Sweden. This climate system was designed to reduce the difference in air temperature between day and night. During the day, humid air was circulated through buried drainage pipes, thus heating the ground and cooling the air. At night, the greenhouse air was heated by the ground. The injection of heat and air into the ground also speeded composting and advanced the start of the growing season. The system has operated successfully since 1987 [6].

The Swiss company Ingenieurbüro Ruess und Hausherr constructed a CI plant where seawater was evaporated in plastic tubes and condensation occurred in buried drainage pipes; this system halved the water consumption of tomato plants [7]. Gustafsson and Lindblom (1999) carried out theoretical and experimental studies on CI in Adana, Turkey. Calculations showed a possible irrigation of 4.6 mm/d with an energy consumption of 1.6 kWh/m³ [8]. Göhlman (1986) studied and performed measurements on the system using both plain and perforated pipes. The resulting heat transfer was 50% higher in the perforated pipes, implying a higher condensation rate [9].

2. Condensation irrigation and drinking water production

The CI system utilises thermal energy to evaporate saline water in, e.g. solar stills. By letting air flow over a warm water surface, the air is warmed and humidified and then led into an underground pipe system where it is cooled and the vapour precipitates as freshwater on the pipe walls. When irrigation is the intended usage, drainage holes in the pipes enable the condensed water and some humid air to leave the pipes and thereby irrigate and aerate the soil [10] (Fig. 1).

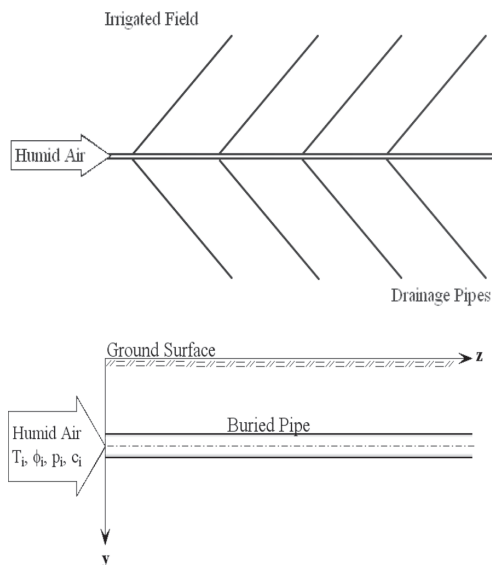


Fig. 1. Plan and section of the pipe system. Warmed and humidified air enters the buried pipes, where it cools and vapour condenses. The humid air properties at the inlet are described by the air temperature, T_a , the air humidity, ϕ , pressure, p , and velocity, c .

Water can be collected at the pipe ending if common pipes are used.

Due to the heat storage capacity of soil, latent and sensible heat from the airflow accumulates in the ground around the pipes, gradually reducing the cooling capacity. To restore the original ground temperature, cold ambient air is injected through the pipe system during the night. Ideally, the heat injection is balanced by an equal amount of cooling. The mean soil temperature will nevertheless be higher than the undisturbed soil temperature during the irrigation season. During fallow periods, the soil temperature is lowered by natural means or by night air cooling.

Evaporation of saline water in solar stills is a common method in producing freshwater. Commercially passive solar stills produce around 3–4 kg/m²/d and are therefore mostly used for small-scale water production [11]. If solar stills were

used to humidify the air, they would be more efficient than ordinary stills since the induced airflow would increase the evaporation rate from the water surface. Although solar stills are suitable for humidifying air to the system, using any other heat source for the evaporation would be possible.

2.1. Outline of a condensation irrigation system

As warm, humid air is cooled while flowing through the pipes, vapour condenses on the cooler pipe walls. Sensible and latent heat from the humid air is transferred from the pipe and accumulates in the surrounding soil. The resulting condensation rate along a pipe depends on the inlet properties of the airflow, soil, and the pipe configuration. To obtain the best irrigation yield, the design of the pipe system and operation must be determined with respect to the local climate and the specified crop.

A soil temperature up to 45°C is favourable for most crops and stimulates root growth, while higher temperatures can be harmful [5,12]. If the pipe temperature rises above this critical temperature the roots will not grow into the pipe and block the airflow. Hence, the system should be designed so that the pipe wall temperature reaches at least 45°C at the end of a daily irrigation. The pipe spacing should be designed so that the soil temperature and the irrigation rate between the pipes are suitable for the crop.

Since soil water is transported to plants by capillary forces, diffusion and root suction, roots can be located at a distance from the pipe and still be able to pick up water. When choosing pipe depth and spacing, these transport mechanisms must be greater than the gravitational forces that draw the water downwards. This implies a shallow pipe location, though for practical reasons the pipes should not be buried too shallow in the field.

2.2. Outline of a system for drinking water production

For drinking water production, common pipes

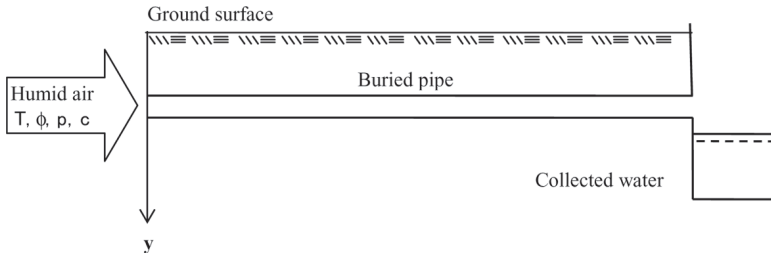


Fig. 2. Outline of the water production pipe system. Heated and humidified air enters the buried pipes, where it cools and vapour condenses. The humid air properties at the inlet are described by the air temperature, T_a , the air humidity, ϕ , pressure, p , and velocity, c .

are used in the system and the condensed water is collected at the pipe endings (Fig. 2). In this case, heat is transferred less efficiently into dry soil because of lower thermal conductivity and negligible mass transport. This results in a lower condensation rate. Previous experiments have shown the heat transfer and thus the condensation rate to be 50% higher in perforated pipes [5].

When drinking water is produced, the condensation rate is not bound to the water production per unit area, but to the total condensation rate along the pipe. The pipe spacing is, hence, chosen with regards to the water need and the available area. The condensation rate decreases with pipe depth, though they should not be buried too shallow to prevent the pipes from being damaged by surface loads.

The maximum soil temperature is less relevant when no crops are to be considered. However, it is the temperature difference between the airflow and the soil that governs the condensation rate of a specific airflow rate. Since the condensation rate is lower in a smooth pipe rather than one that is perforated, the daily period of water condensation can be extended longer.

3. Mathematical model

This study analyzes the potential for drinking water production in the CI system and is thus restricted to the condensation of water in buried

pipes that assume certain inlet air conditions, such as pressure, velocity, temperature and humidity. The difference between the applications of irrigation and drinking water is that the former uses drainage pipes while the latter involves common pipes. Consequently, air and water may only leave through the pipe outlet in the drinking water application, and thereby negate any mass transport in the soil.

3.1. Humid airflow properties

Given the inlet airflow conditions, the changing properties of the air are determined numerically along a pipe by approximating both air and vapour as ideal gases. The characteristics of a humid airflow through a control volume in the pipe from location i to $i+1$ can be estimated by:

$$q = \dot{m}_i \cdot \left[T_{a,i} \cdot c_{v,i} + \frac{L_i \cdot w_i}{1 + w_i} + \frac{p_i}{\rho_i} + \frac{c_i^2}{2} \right] + \dot{m}_{w,i} c_p T_i - \dot{m}_{i+1} \cdot \left[T_{a,i+1} \cdot c_{v,i+1} + \frac{L_{i+1} \cdot w_{i+1}}{1 + w_{i+1}} + \frac{p_{i+1}}{\rho_{i+1}} + \frac{c_{i+1}^2}{2} \right] - \dot{m}_{w,i+1} c_p T_{i+1} \quad (1)$$

At the inlet, i , and the outlet, $i+1$, $\dot{m}$ is the mass flow rate of humid air, T_a is the fluid temperature, c_v is volumetric specific heat, w is absolute

humidity, p is pressure, ρ is density, c is velocity and L is the latent heat of vaporization. The airflow rate is reduced by the condensation rate while flowing through the control volume. The resulting heat transfer, q , from the pipe wall to the surrounding soil is the sum of the convective heat transfer, q_{conv} , and the latent heat release from condensation, q_{cond} :

$$q_{\text{conv}} + q_{\text{cond}} = -k \nabla T \cdot \hat{n} \quad (2)$$

where $\nabla T \cdot \hat{n}$ is the temperature gradient in the normal direction to the pipe wall and k is the thermal conductivity of the soil. The convection heat transfer from the pipe q_{conv} is

$$q_{\text{conv}} = h \cdot (T_a - T_p) \quad (3)$$

where T_p is the pipe wall temperature and h is the thermal convection coefficient, which is a function of the flow properties and the thermal conductivity of air, k_a . The Nusselt number (Nu) for constant heat flux is here a function of the friction coefficient, f , and the Reynold's (Re) and Prandtl (Pr) numbers [13]:

$$\text{Nu} = \frac{h \cdot D}{k_a} = \frac{\left(\frac{f}{8}\right) \cdot (\text{Re} - 1000) \cdot \text{Pr}}{1 + 12.7 \cdot \left(\frac{f}{8}\right)^{0.5} \cdot (\text{Pr}^{2/3} - 1)} \quad (4)$$

where the friction coefficient, f , is

$$f = (0.79 \cdot \ln \text{Re} - 1.64)^{-2} \quad (5)$$

As vapour condenses in the pipe its latent heat, q_{cond} , is released:

$$q_{\text{cond}} = L \cdot h_m \cdot (\phi \cdot \rho_{v,s}(T_a) - \rho_{v,s}(T_p)) \quad (6)$$

L is the latent heat of vaporization of water, ϕ is the relative humidity, $\rho_{v,s}(T_a)$ is the saturated vapour density at airflow temperature, $\rho_{v,s}(T_p)$ is

the saturated vapour density at pipe wall temperature and h_m is the mass convection coefficient, which is analogous with the thermal convection coefficient [13] according to

$$h_m = \frac{h \cdot \alpha^{1/3} \cdot D_{AB}^{2/3}}{k_a} \quad (7)$$

where α is the thermal diffusion coefficient of the soil and D_{AB} is the binary diffusion coefficient of air–water vapour. The saturation pressure of vapour, $p_{v,s}$, is approximated according to [14]:

$$\rho_{v,s} = \frac{e^{\left(\frac{77.345 + 0.0057(T_a + 273.15)}{T_a + 273.15} - \frac{7235}{T_a + 273.15}\right)}}{(T_a + 273.15)^{8.2}} \quad (8)$$

Since no air leaves the pipe, its mass flow of dry air is constant throughout the pipe. A change in mass flow rate occurs only in the vapour, which decreases as it condenses. The mass flow of liquid water is assumed to be negligible. While passing through the control volume, the vapour flow rate is reduced by $\dot{m}_c$:

$$\dot{m}_c = \pi \cdot D \cdot \Delta z \frac{q_{\text{cond}}}{L} \quad (9)$$

where Δz is the distance between location i and $i + 1$ along the pipe. The humidity of the airflow also decreases when the vapour condenses, and the absolute humidity at location $i + 1$ in the control volume, w_{i+1} , becomes

$$w_{i+1} = w_i - \frac{\dot{m}_c}{\dot{m}_a} \quad (10)$$

3.2. Heat transfer in the soil

During water production, energy transferred from the airflow, q , increases the pipe wall temperature, from where it is conducted as heat through the soil. During nightly cooling, some of the accu-

mulated heat in the soil is transferred back into the cooler airflow. For constant thermal properties of the soil, the transient heat equation becomes:

$$\frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{\partial T}{\partial z} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (11)$$

3.3. Simulations

To analyze the water production potential of the system, an explicit 3D FDM-model was developed in MATLAB 6.5 using the theory previously described. The simulated system operated in a climate where the diurnal ambient air temperature, T_{aa} , varied according to

$$T_{aa} = 8 \cdot \sin \left(\frac{\Delta t \cdot 2\pi \cdot n}{24 \cdot 3600} \right) + 20 \quad (12)$$

where n is the number of time steps, Δt . The underground condensation process occurred during the daily 12-h operation when the ambient temperature was warmer than 20°C. The rest of the day was devoted to ground cooling by pumping ambient air directly through the simulated pipe. As a reference case, inlet parameters and pipe configuration were chosen according to Table 1.

Since the mean temperature of the ambient air in the presumed climate is 20°C, this temperature was also chosen as the undisturbed soil tempera-

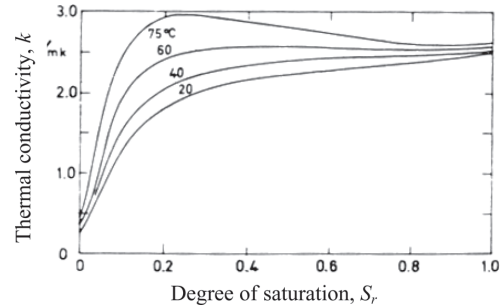


Fig. 3. Temperature and humidity dependence of thermal conductivity in moist sand [15].

ture. At a 7.5 m depth, the soil was assumed to have a constant temperature equal to the mean ambient air temperature. The thermal conductivity of the soil was derived from Fig. 3 for 25% saturation to $k = 2$ W/m K.

The numerical simulations were performed using a cell size of 0.1 m in the x - and y -directions and 1.0 m in the z -direction, i.e. along the pipe. The boundary conditions used in the model were:

$$\begin{aligned} x = -X/2, \quad x = X/2 & \quad \frac{\partial T}{\partial x} = 0 \\ y = Y & \quad T = 20 \\ z = 0 \text{ and } z = Z & \quad \frac{\partial T}{\partial z} = 0 \end{aligned}$$

where Z is the vertical section at the pipe outlet, Y is the horizontal section at depth 7.5 m, and X is the pipe spacing.

The energy content of the condensed water is very small (<1%) compared to the total heat transfer and is therefore neglected in the performed calculations. Although the condensate is the ultimate product of this system, the mass and volume of the water are assumed to have no effect on the airflow. In effect, this can be seen as if the condensate was removed when formed. The pipe walls are nevertheless considered to be wet.

Table 1

Inlet air parameters and pipe properties of the reference case

Temperature, $T_{a,in}$, °C	60
Relative humidity, ϕ_{in} , %	70
Velocity, c_{in} , m/s	3.5
Pressure, p_{in} , atm	1
Pipe depth, d , m	0.5
Pipe spacing, cc , m	1
Pipe diameter, D , m	0.2

4. Results and discussion

The condensation rate in the prescribed reference system decreases rapidly during the first few days and reaches a constant daily level after about 90 days of operation. This decrease in water production is a result of the increased ground temperature around the pipe. Fig. 4 illustrates the mean condensation rate of one pipe during the first 90 days of operation. During the first month of operation, the water production was 2.3 kg/m d per pipe, while the following month yielded 1.9 kg/m d. An increased daily cooling time would mean less time for condensation, i.e. the slope of the curve in Fig. 4 would become less steep.

Water production during the first 90 d of operation is mainly relevant for the CI application, since the usual duration of one irrigation period ranges from 3 to 6 months. The drinking water system is meant to run continuously; so the steady state production level is used for dimensioning a system. Therefore, the analysis of the simulation model was performed during the 91st and 92nd days, as the total hourly condensation varied according to Fig. 5. The resulting total condensation rate per day yields 89.3 kg/d, i.e. approximately 1.8 kg/m/d.

The total hourly condensation rate decreases by about 40% during the daytime (from 12.1 kg/h

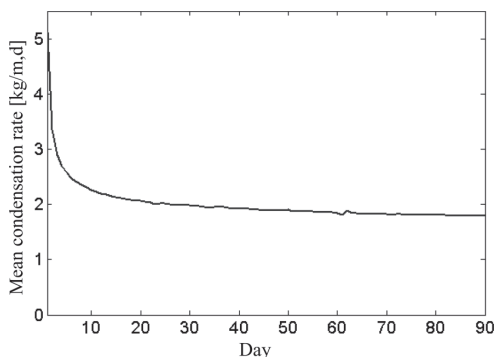


Fig. 4. Calculated daily mean condensation rate in one pipe per unit length during the first 90 days.

to 7.4 kg/h) due to the heating of the ground. However, since the system has reached steady state, the stored thermal energy from one day of condensation is completely balanced by nightly cooling.

As the air becomes cooler and drier while flowing inside the pipe, the local condensation rate also decreases along the pipe. Hence, short pipes yield a higher mean water production rate along the entire length, though the total production becomes less. The accumulated condensation rate along the pipe is shown in Fig. 6 for different spacings.

For the reference inlet parameters, pipe depth and diameter, the condensation yield, $C(z)$, is influenced by the pipe length and spacing according to:

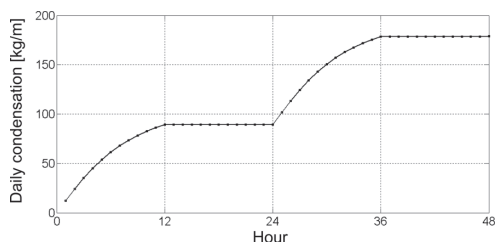


Fig. 5. Calculated total water production per hour and pipe during the 91st and 92nd day.

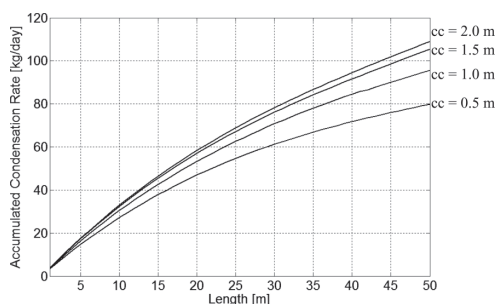


Fig. 6. Calculated accumulated water production per pipe in a system with different spacing.

$$C(z) = (k_0 + k_1 z + k_2 z^2 + k_3 z^3) \cdot 10^{-6} \quad (13)$$

where

$$k_0(cc) = -436,666.62 + 1,677,411.06cc \\ -1,471,937.92cc^2 + 381,431.97cc^3$$

$$k_1(cc) = -2,116,570.06 + 1,150,883.21cc \\ -455,992.30cc^2 + 63,347.38cc^3$$

$$k_2(cc) = -40,040.11 + 32,442.11cc \\ -27,661.22cc^2 + 7088.70cc^3$$

$$k_3(cc) = 407.21 - 786.52cc + 670.64cc^2 \\ -171.00cc^3$$

for $1 \leq z \leq 50$ and $0.2 \leq cc \leq 2.0$. A pipe length of 50 m with a 1.0 m spacing yields the same amount

of water as 40-m long pipes with a 2.0-m spacing, though the area required in the latter case is 60% greater. Therefore, the highest water production in this system is obtained when the pipes are placed as narrow as possible. The pipes should also be made as short as the area allows.

The sectional temperature distribution for the first and last hour of irrigation during the 91st day is shown in Fig. 7.

The pipe is located at the centre of each section at 0.5-m depth. The temperature close to the pipe changes considerably from the first hour of irrigation to the last, while the temperature farther away from the pipe is more or less constant. A steady temperature level during daily operation, located about 0.15 m above the pipe and about 1.0 m below, is reached at different distances from the pipe because the ambient air contributes by heating the topsoil during the day and cooling during the night. The soil temperature above the pipe quickly reaches a steady level between the

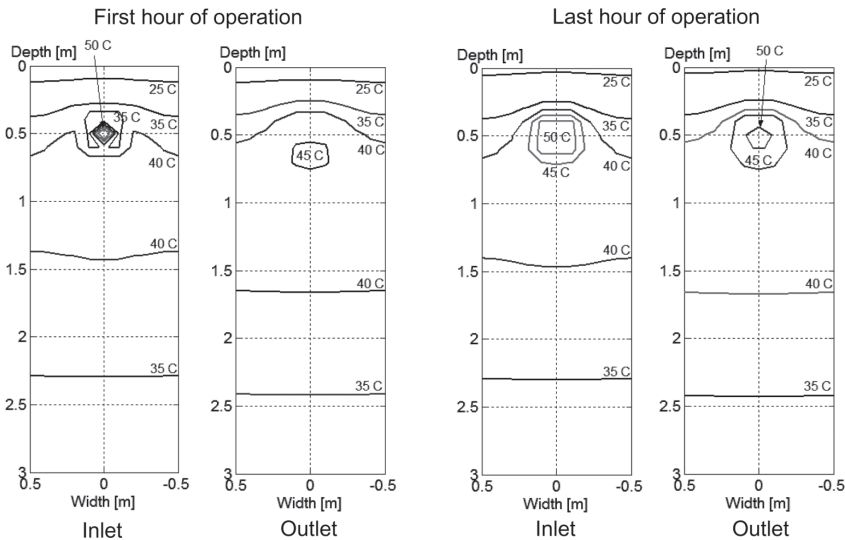


Fig. 7. Calculated temperature distribution around the pipe in sections at the pipe inlet and outlet, during the first and last hour during daytime operation.

surface and the pipe, while the distance to a steady temperature under the pipe is considerably greater.

Again, the critical root temperature (45°C) is decisive in the operation of a CI system. Of note that in this system, the simulated temperature between two pipes varies between 35°C and 40°C, with temperatures of 45°C and above being restricted to the immediate area around the pipe. Hence, for this temperature distribution, cultivation would be possible.

The soil temperature along the pipe is shown in Fig. 8, where the pipe with a diameter of 0.2 m is located at 0.5-m depth.

The mean soil temperature below the pipe increases with its length due to the mean temperature of the airflow being greater at the outlet than the inlet ($\bar{T}_{in} = 37.5^\circ\text{C}$ and $\bar{T}_{out} = 46.3^\circ\text{C}$).

The heat transferred to and from the airflow inside a buried pipe is plotted for the 91st day in Fig. 9. The gradual decrease in heat flow is a result

of a higher condensation rate at the beginning of the day. As the soil closest to the pipe is heated during daytime operation, the condensation process slows down. Towards the end of the day, the soil temperature along the pipe evens out, and the heat flow stabilises.

A sensitivity analysis was performed to analyse the influence of different parameters on system performance. Selected inlet parameters were altered $\pm 20\%$ and the daily condensation rate was evaluated. This analysis, conducted on the 91st day of operation, was then compared with the corresponding value of the reference system (Fig. 10).

Fig. 10 indicates air temperature and humidity as being the most dominating factors influencing water production. A change in either of these parameters gives a correspondingly greater change in the condensation rate.

The energy required to transport the air through a pipe can be approximated as the pressure change

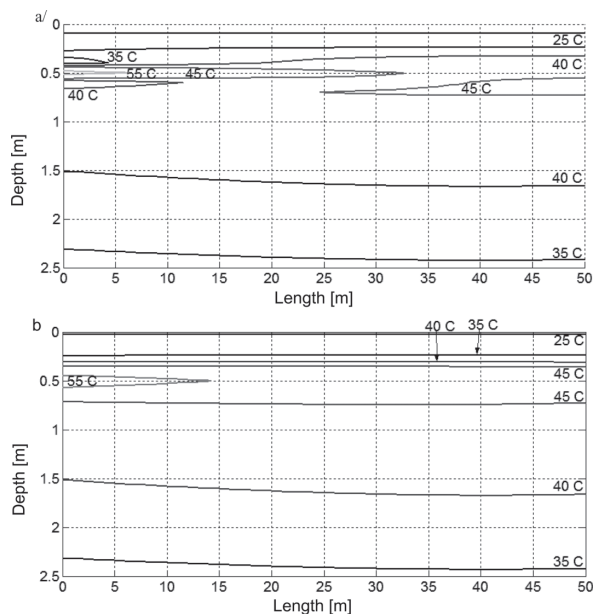


Fig. 8. Calculated temperature distribution in a section along the pipe during a) the first and b) the last hour of daytime steady-state operation.

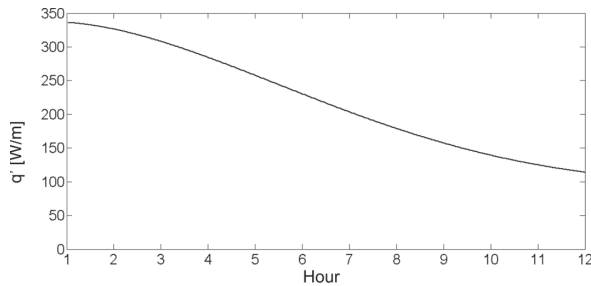


Fig. 9. Calculated heat flux from the pipe during the 91st day (steady-state operation).

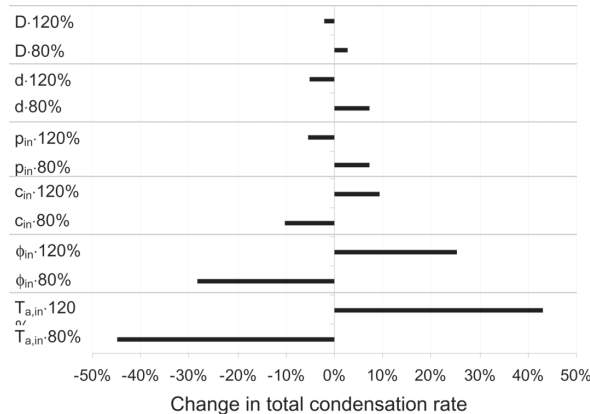


Fig. 10. Sensitivity analysis of the inlet parameters air temperature (T_i), relative humidity (ϕ_i) air velocity (c_i) air pressure (p_i), pipe diameter (D) and depth below ground surface (d).

due to wall friction and airflow acceleration (or retardation). During daytime operation (air cooling), density increases along the pipe and the loss in acceleration pressure consequently becomes a pressure gain. For the reference condition, the energy required for one day of operation, Q_{fan} , is:

$$Q_{\text{fan}} = \frac{\bar{m}}{\rho} \cdot \left[\bar{p} \cdot \bar{f} \cdot \frac{L}{D} \cdot \frac{\bar{c}^2}{2} + \rho_{\text{in}} \cdot c_{\text{in}}^2 \cdot \left(\frac{\rho_{\text{in}}}{\rho_{\text{out}}} - 1 \right) \right] \cdot 24 \quad (14)$$

where the average airflow is $\bar{m} = 0.108 \text{ kg/s}$ and the velocity is $\bar{c} = 3.40 \text{ m/s}$ between pipe inlet and outlet, $c_{\text{in}} = 3.48 \text{ m/s}$, $\rho_{\text{in}} = 1.005 \text{ kg/m}^3$, $\rho_{\text{out}} = 1.022 \text{ kg/m}^3$ and $\bar{f} = 0.023$, i.e. the average friction factor obtained from Eq. (5).

The resulting fan energy from 24 h of operation is 85.6 Wh and since the reference case resulted in 89.3 kg of desalinated water, the running cost measured in driving energy becomes 0.96 kWh/m^3 of produced water.

Crops usually need 400–800 mm of water per year [8], though this varies during the dry and

wet periods. For example, the average water need for tomatoes is 130 mm/month [16]. Experiments performed by Hausherr and Ruess indicated a 50% reduction in the water needed for tomato plants by using sub-irrigation with humid air [7]. Assuming that this reduction is generally valid the mean water demand of tomatoes would be reduced to 65 mm/month, i.e. 2.2 mm/d. The simulated water production system delivers at least this amount of water in the first 15 m of the pipe (reference system), though the CI system would yield considerably more water. It should be noted that tomato plants required more water than most plants.

5. Conclusion

In the described system for irrigation or drinking water production humid air was condensed while flowing through buried pipes in the ground. For irrigation to occur, the pipes must be perforated. This project focussed on drinking water production where common pipes were used to collect the condensed water at the pipe endings.

The cooling capacity of the ground was sustained by night time cooling. The simulated reference case gave a daily mean water production of 1.8 kg per 1 m of pipe. Water production was enhanced by increasing air temperature, humidity and velocity, or by reducing pipe length, spacing, depth and air pressure.

The mean ambient air temperature of the simulations was 20°C. In a warmer climate, the nightly cooling would be less efficient and the mean soil temperature would be higher, leading to reduced water production. However, the inlet air temperature would be higher and carry more vapour into the ground, and somewhat compensate for the warmer climate.

The mean air temperature, after heating and cooling, was higher at the outlet than the inlet, meaning that the ground temperature around the pipe was higher at the pipe ending. This indicates that a longer ground cooling interval or a higher

night air velocity would yield a higher condensation rate for the pipe.

The simulated system, designed for drinking water production, could satisfy crop water needs, and the occurring soil temperature would not hinder crop root development. If perforated pipes had been used, the surrounding ground would have experienced more rapid temperature changes, and the condensation process would have been more effective.

The calculated fan energy to drive air through one straight pipe was 0.96 kWh/m³ water. This energy cost would be reduced with an increased inlet air temperature.

6. Future work

Continued analyses of the CI system include how pipe perforations would affect the mass and heat transfer in the ground. The heat transfer from the pipe into the soil is expected to increase by 50% when drainage pipes are used [9], resulting in a correspondingly greater condensation rate. Experimental and numerical studies will be performed to fully understand this complex mass and heat transport.

Planned field tests involve both plain and perforated pipes. The measured performance will be compared with simulations to validate a model to be used in designing future CI plants.

Extended studies will examine effects of pre-heating of the air or water in the solar stills. Different means of transporting the warm, humid air will also be considered to reduce investment costs. Different solar still constructions will be investigated to estimate the required area for the humidification process. A solar driven CI system will be outlined to estimate the total irrigation cost.

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Symbols

C	— Accumulated condensation rate, kg/d
c	— Velocity, m/s
cc	— Pipe spacing, m
c_p	— Specific heat capacity at constant pressure, J/kg, °C
c_v	— Specific heat capacity at constant volume, J/kg, °C
d	— Pipe depth, m
D	— Pipe diameter, m
D_{AB}	— Diffusion coefficient of air–vapour, m ² /s
f	— Friction factor
h	— Thermal convection coefficient, W/m ² , °C
h_m	— Mass convection coefficient, m/s
k	— Thermal conductivity, W/m, °C
L	— Latent heat of fusion, J/kg
m	— Mass, kg
n	— Number of time steps
Nu	— Nusselt number
p	— Pressure, Pa
Pr	— Prandtl number
Q	— Energy, J
q	— Heat flux, W/m ²
Re	— Reynold number
T	— Temperature, °C
t	— Time, s
w	— Absolute air humidity, kg vapour/kg dry air
x	— Horizontal direction across the pipes
y	— Vertical direction
z	— Horizontal direction along the pipes
α	— Thermal diffusivity, m ² /s
ϕ	— Relative humidity
ρ	— Density, kg/m ³

Subscripts

a	— Air
aa	— Ambient air
c	— Condensed water
cond	— Condensation

conv	— Convection
f	— Fluid
fan	— Fan
i	— Node position in y-direction
in	— Inlet
j	— Node position in z-direction
k	— Node position in x-direction
out	— Outlet
p	— Pipe
s	— Saturated
v	— Vapour
w	— Liquid water

Superscripts

$\cdot$	— Per second
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Paper II

Underground production of Humid air for drinking water production and subsurface irrigation

Lindblom, J., Nordell, B.

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Underground condensation of humid air for drinking water production and subsurface irrigation

Jenny Lindblom* and Bo Nordell

Environmental and Civil Engineering, Luleå University of Technology, SE-97187 Luleå, Sweden
Tel: +46 920 491330; Fax: +46 920 491697; email: jeli@ltu.se

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Abstract

Condensation Irrigation (CI) is a combined system for solar desalination and irrigation and/or drinking water production. Solar stills are used for humidifying ambient air flowing over the saline water surface in the still. This warm, humid air is then led into an underground pipe system where it is cooled and vapour precipitates as freshwater on the pipe walls. If drainage pipes are used the condensed water and some of humid air percolate through the pipe perforations and irrigates and aerates the ground. Drinking water can be collected at the pipe endings when using non-perforated pipes. The CI system has attracted attention from several North African countries, and pilot plants are now in operation in Tunisia and Algeria. Mass and heat transfer in the soil around the buried pipes has been modelled to evaluate the theoretical potential for these types of systems and to gain understanding of the mechanisms governing their productivity. For a presumed reference system, the mean water production rate in the drinking water system was 1.8 kg per meter of pipe and day. When using drainage pipes for subsurface irrigation, this number increased to 3.1 kg/m/d, corresponding to 2.3 mm/d of supplied irrigation water.

Keywords: Desalination; Irrigation; Water production; Heat and mass transfer; Modelling

1. Introduction

Ever since the Industrial Revolution, the world's population has grown at unprecedented rates due to our improved standard

of living. Meeting the increasing demand for food and water has become one of the most urgent dilemmas for both humans and the environment [1]. However, the fear one day

*Corresponding author.

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of a population exceeding the food supply has so far proved untrue: modern technology and mechanization in agriculture have managed to keep pace.

During the Green Revolution starting in the 1960s, the extensive use of fertilizers, expansion of irrigation and the introduction of high-yielding crops averted the global famine of the former century [2]. As a result, global grain output grew by 170% from 1% more land area and saved approximately 1 billion lives in the years between 1950 and 1992 [3].

As the number of people continues to grow, the United Nations projects that nearly 8 billion humans will inhabit the Earth by 2025, among which 80% will live in developing countries [4]. To grow food for this escalating population, agriculture production must grow at a corresponding rate on a more or less constant land and water resource base.

However, the past successes of the Green Revolution will not be easily repeated. The areas benefiting the most from Green Revolution technology were fertile and rich in freshwater. The countries predicted to risk famine this time will be semi-arid and arid regions in the developing world where the most degraded lands are found and freshwater is scarce [2].

The most limiting factor for crop cultivation is water availability in the rooting zone [5]; with secure access to irrigation water harvests can be increased by up to six times [6]. But the potential for expanding irrigation remains limited: the water resources in many regions have been fully exploited, and the competition for water resources between agricultural, industrial and domestic uses grows continuously [7].

About 72% of withdrawn freshwater is used in agriculture, a figure that approaches 90% in irrigation intensive and arid regions [6]. The Consultative Group on International Agricultural Research (CGIAR) states that

even if everything is done to make irrigated agriculture more water efficient, humanity will still need at least 17% more freshwater to meet all of its future food needs [2].

The only sustainable way to produce more freshwater is through desalination of seawater. In oil rich countries such as Kuwait, Qatar, Bahrain, Saudi Arabia, and the United Arab Emirates, about 95% of all freshwater is already supplied by desalination technologies using fossil fuels [8]. In view of future oil shortages, desalination must, however, be driven with renewable energy.

The Condensation Irrigation (CI) system combines desalination with irrigation by humidifying ambient air with saline water in, e.g. solar stills and dehumidifying it in drainage pipes buried in the ground. Condensed water and humid air infiltrate the soil through the pipe perforations and irrigate and aerate the soil. Subsurface irrigation increases the irrigation efficiency and eliminates water loss through surface runoff.

As per daily low-flow water distribution, the risk of land degradation is reduced as salt transport to the topsoil via a rising groundwater table is prevented. Furthermore, when using solar driven air humidification, water production is more effective during warm and sunny days when the irrigation need is high. The CI technology can also be applied to drinking water production through non-perforated pipes in the ground. The condensed water from the humid airflow can then be collected at the pipe endings.

Work concerning the Condensation Irrigation system started at the Luleå University of Technology (LTU) as a series of Masters Theses [9–11], and the technology was used in the construction of a greenhouse climate control facility in Övertorneå, Sweden [12].

Independently of these studies, the Swiss company Ingenieurbüro Ruess und Hausherr constructed a CI plant where seawater had

evaporated in plastic tubes, with the condensation occurring in buried drainage pipes. A reported 50% reduction in the water consumption of tomato plants was observed in the system [13].

2. Condensation irrigation

The Condensation Irrigation (CI) system uses thermal energy to evaporate saline, or otherwise polluted water, in e.g. solar stills. Ambient air is humidified by the warm water inside the still and thereafter led into an underground pipe system where it is cooled and the vapour precipitates as freshwater.

When drainage pipes are buried in the ground, condensed water and some humid air leave the pipes through the perforations and thereby irrigate and aerate the soil in the rooting zone. By using non-perforated pipes the condensed water from the pipes can be collected at the pipe endings and used for drinking or other purposes. The principle of the CI system is shown in Fig. 1.

As the airflow is dehumidified in the pipes the surrounding soil gradually becomes heated, thereby reducing the water production efficiency. To decrease the ground temperature, cold ambient air is circulated through the pipe system during the night. The mean soil temperature will nevertheless be higher than the undisturbed soil temperature during irrigation periods. When irrigation is not needed, the soil temperature is

lowered naturally or by the auxiliary ambient air cooling.

When designing the underground condensation pipe configuration, climate, soil type, and expected inlet humid airflow properties, must all be considered. If the system is intended for irrigation, crop selection must also be considered in terms of water need, rooting depth, leaf area, ability to take up water, temperature sensitivity of roots, etc.

Although requiring larger land areas, using wide spacing between the pipes results in higher condensation rates and lower installation costs. This will also lead to lower soil humidity and ground surface evaporation rates for irrigation applications. Pipe lengths affect the air dehumidification efficiency, since the condensation rate decreases along the pipe [14–15].

Shallow pipe depths may increase the pipe condensation rate and result in a shallow soil water distribution that accumulates above the pipe depth. When considering solar radiation, a shallow depth could on the contrary lead to less water production and higher surface evaporation due to additional heating of the ground surface.

Commercial passive solar stills produce about 3–4 kg/m²/d and are therefore mostly used for small-scale water production [16]. Using solar stills to humidify the air in the CI system might be more efficient than ordinary stills since the airflow would increase the evaporation rate from the water surface.

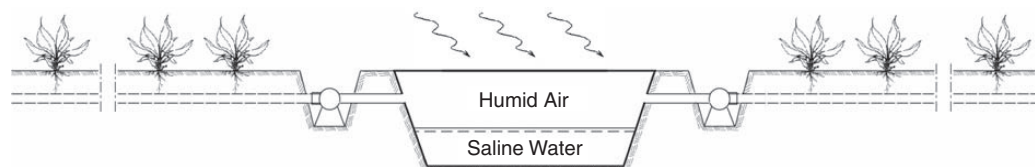


Fig. 1. Outline of a condensation irrigation system. Ambient air is warmed and humidified inside solar stills and led into buried pipes, where it is cooled and dehumidified.

Although solar stills are suitable for humidifying air in the CI system, other methods and heat sources are possible depending on different site-specific conditions. A solar driven alternative would be to use existing lined irrigation canals that are impermeable to salts. Converting these to air humidification canals would require a solar radiation transparent cover and possibly a radiation absorbing material onto the canal borders.

2.1. Drinking water production

By dehumidifying the warm airflow inside buried non-perforated pipes, freshwater can be collected at the pipe endings and used for drinking. The study on this system was done to analyse the pipe hydraulics and the transient heat transfer and accumulation in the surrounding ground, without having to consider the mass transport in the soil.

Nevertheless, the CI system for drinking water production in rural areas could enhance the efficiency compared to ordinary solar stills. This is partly due to the increased heat and mass transfer of the air stream over the saline water surface in the still that could increase the evaporation rate, and to the external dehumidification that increases the condensation.

2.2. Subsurface irrigation

In the subsurface condensation irrigation system, buried drainage pipes are used to conduct the humid air. The pipe perforations enable air and water to infiltrate the surrounding soil thereby irrigating the crop directly in the rooting zone.

The mass transport in the ground contributes to an increased heat flux from the pipe, improving the air dehumidification compared to drinking water production systems. This also implies a lower ground temperature. The type of crops will also influence the soil

temperature and moisture distribution differently, since the root development and water uptake ability of crops are somewhat temperature dependent.

For most cultivated crops, temperatures up to 45 °C are considered stimulating for root growth, while higher temperatures can be harmful [9]. In the CI system, the pipe configuration should therefore be such that this critical temperature is reached at the pipe walls, but never in the soil between two pipes. In so doing, plant roots are free to develop in-between the pipes, but not into the pipes, where they block the airflow (Fig. 2).

Root intrusion is one of the major problems in subsurface drip irrigation systems, and is usually solved by injecting acid through the pipes [17]. In the CI system, this can hence be avoided through high airflow temperatures.

There are many advantages with using subsurface irrigation. Especially worth mentioning

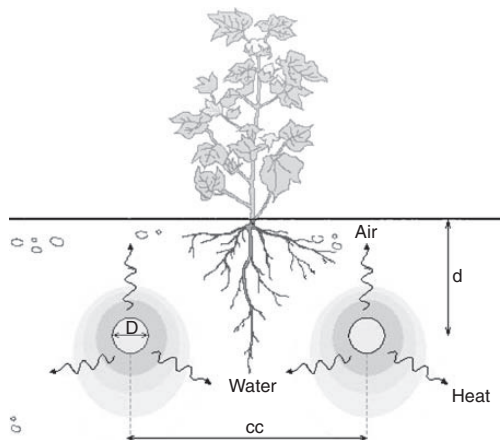


Fig. 2. Section of buried drainage pipes buried in a field. Heat, condensed water and humid air flow through the pipe perforations into the soil. The pipe diameter D , depth d , and spacing cc are here denoted.

is that the water use becomes more efficient, surface losses from evaporation and run-off is lowered, harvesting becomes easier, and surface crusts are prevented. Among the drawbacks experienced from using a sub-surface scheme is that deep ploughing is prevented, cultivation is restricted to specified row spacing, and potential root intrusion might complicate the irrigation. Crops successfully cultivated with subsurface irrigation are many. Some of them are presented in Table 1 with their typical irrigation need [18–19]:

3. Simulation results and discussion

Theoretical simulations on systems for drinking water production and for subsurface irrigation have been performed to examine the potential for using the CI technology. The two models used arrays of 50 m long pipes, spaced 1.0 m apart and buried at 0.5 m depth [14–15].

The climate in the models was assumed to be free from solar radiation and precipitation. Both factors could affect water production in the pipes, since solar radiation would add to the heating of the ground surface, and thus lower the condensation rate, and precipitation would have a positive effect by cooling the ground.

At the soil surface, the heat transfer was affected by the ambient air with a constant convection coefficient of $12 \text{ W/m}^2 \text{ K}$ and a

temperature that varied from 12 to 28°C according to:

$$T_{aa} = 8 \cdot \sin\left(\frac{\Delta t \cdot 2\pi \cdot n}{24 \cdot 3600}\right) + 20 \quad (1)$$

During the day's 12 hours when the ambient air temperature was at 20°C or warmer, water was produced by letting a warm, humid airflow enter the pipes with constant inlet properties given in Table 2.

The rest of the day was devoted to nightly ground cooling, and ambient air at varying temperatures according to Eq. (1) was circulated through the pipes. In reality, the inlet properties during the daily water production would vary with the climate from day to day and from hour to hour, mainly depending on the solar radiation.

The transient heat and mass transfer in the soil-pipe system was simulated for a period of three months, after when the condensation rate was constant from one day to another in both models.

3.1. Simulations of the drinking water production system

By using non-perforated pipes the total mass of the condensed water and humid air is preserved while flowing through the pipe. Neglecting frictional losses, the airflow properties along the pipe and the heat flux to the surrounding soil may be presented according to

Table 1

Crops used in sub-surface irrigation and their required irrigation

Tomato	130	mm/month
Cotton	100–130	mm/month
Onion	100	mm/month
Sorghum	300–650	mm/harvest
Beans	250–500	mm/harvest

Table 2

Reference system properties

Temperature, $T_{av,in}$	60	$^\circ\text{C}$
Relative humidity, $\phi_{av,in}$	70	%
Velocity, $c_{av,in}$	3.5	m/s
Pipe depth, d	0.5	m
Pipe spacing, cc	1.0	m
Pipe diameter, D	0.2	m

$$\begin{aligned}
 q = \dot{m}_i \cdot & \left[T_{a,i} \cdot c_{v,i} + \frac{L_i \cdot w_i}{1 + w_i} + \frac{p_i}{\rho_i} + \frac{c_i^2}{2} \right] \\
 & + \dot{m}_{w,i} c_p T_i - \dot{m}_{i+1} \\
 & \cdot \left[T_{a,i+1} \cdot c_{v,i+1} + \frac{L_{i+1} \cdot w_{i+1}}{1 + w_{i+1}} + \frac{p_{i+1}}{\rho_{i+1}} + \frac{c_{i+1}^2}{2} \right] \\
 & - \dot{m}_{w,i+1} c_p T_{i+1}
 \end{aligned} \quad (2)$$

At the inlet, i , and the outlet, $i + 1$, is the mass flux of humid air, T_a is the fluid temperature, c_v is volumetric specific heat, w is absolute humidity, p is pressure, ρ is density, c is velocity and L is the latent heat of vaporization. While flowing through the control volume the mass flux of humid air is reduced by the mass of condensed water.

The enthalpy of the condensed water was found to be very small (<1%) compared to the total heat released and was therefore neglected in the calculations. In effect, this can be seen as if the condensate was removed when formed. The pipe walls were nevertheless considered to be wet.

The resulting heat transfer, q , from the pipe wall to the surrounding soil is the sum of the convective heat transfer and the latent heat release from condensation:

$$\begin{aligned}
 q &= h \cdot (T_a - T_p) + L \cdot h_m \\
 &\quad \cdot (\phi \cdot \rho_{v,s}(T_a) - \rho_{v,s}(T_p)) \\
 &= -k \nabla T \cdot \hat{n}
 \end{aligned} \quad (3)$$

where $\nabla T \cdot \hat{n}$ is the temperature gradient in the normal direction to the pipe wall and k is the thermal conductivity of the assumed uniform soil. Since the mass transport may be neglected in the soil from a lack of pipe perforations, the heat transfer in the ground becomes a pure conduction problem, written as:

$$\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (4)$$

where α is the thermal diffusion coefficient of the soil. As shown in Fig. 3, the condensation rate in the pipe decreases drastically during the first 10 days, evens out after about 1 month and then reaches a diurnal steady water production rate of 1.8 kg/m/d for a 50 m long pipe.

The decreasing condensation rate inside the pipes is due to the gradual heating of the surrounding soil. If the nightly cooling had been able to remove all excess heat released from the pipes during the day, this reduction would not have occurred. Also, by enhancing the cooling, e.g. by prolonging the diurnal cooling period or increasing the nightly airflow velocity, a steady water production would be attained faster and level out at higher production rates.

The temperature distribution in the mid-section of the soil-pipe system on the 91st day of operation is shown in Fig. 4 during the 6th and 18th hour, i.e. at midday and midnight. During day the temperature inside the pipe is above 45°C and the temperature in the top 0.5 m soil layer is roughly above 25°C. The soil at greater depths than the pipe is mainly warmer than 40°C. At night the soil is cooled from above and from the cool airflow inside the pipes.

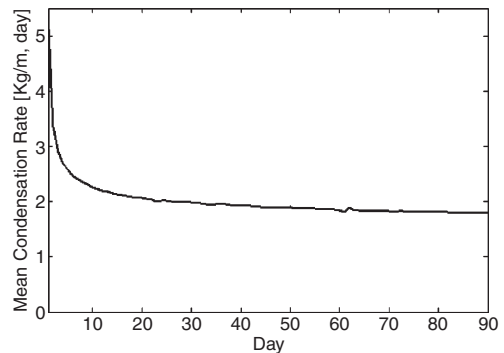


Fig. 3. Daily mean condensation rate per pipe and unit length during the first 90 days.

As the temperature difference between the airflow and pipe walls declines along the pipe, the condensation rate decreases with pipe length. For the reference inlet airflow properties and pipe depth, the condensation yield, $C(z)$, is influenced by the pipe length and spacing according to:

$$C(z) = (k_0 + k_1z + k_2z^2 + k_3z^3) \cdot 10^{-6} \quad (5)$$

where

$$\begin{aligned} k_0(cc) &= -436666.62 + 1677411.06cc \\ &\quad - 1\,471937.92cc^2 + 381431.97cc^3 \\ k_1(cc) &= -2116570.06 + 1150883.21cc \\ &\quad - 455992.30cc^2 + 63347.38cc^3 \\ k_2(cc) &= -40040.11 + 32442.11cc \\ &\quad - 27661.22cc^2 + 7088.70cc^3 \\ k_3(cc) &= 407.21 - 786.52cc + 670.64cc^2 \\ &\quad - 171.00cc^3 \end{aligned}$$

for $1 \leq z \leq 50$ and $0.2 \leq cc \leq 2.0$. The relationship in Eq. (5) is plotted in Fig. 5.

From the relation between pipe spacing and condensation rate inside pipes of various lengths, it can be stated that a narrow spacing and short pipe lengths yield the highest condensation rate per unit ground surface area. The highest water production per unit length of pipe is however obtained by placing the pipes wide apart, since the condensation rate

increases with the pipe distance up to 2.2 m spacing.

When comparing how the inlet airflow parameters affect the steady state condensation rate it was found that the air temperature and humidity were the parameters of highest importance. Table 3 shows the change in condensation rate (as a percentage of the deviation from the reference value) for a change in selected inlet parameters value of $\pm 20\%$.

The timing and length of the water producing and night cooling operations also affect the resulting condensation rate. A maximum condensation rate of 1.9 kg/m/d was obtained when the nightly cooling period increased from 12 to 16 hrs.

3.2. Simulations of the subsurface condensation irrigation system

By using drainage pipes for the subsurface air dehumidification, the ground is irrigated by the water and humid air that infiltrates the ground through the pipe perforations.

The energy and mass conservation equations of the pipe airflow from Eq. (2–3) were in the present model complemented with terms for representing the humid air flux through the perforations.

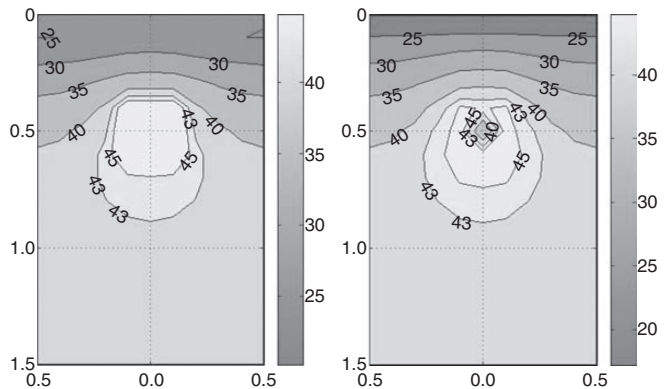


Fig. 4. Temperature distribution in the midsection of the buried non-perforated pipe during the 6th hour of a) water production and b) night cooling on the 91st day. The centre of the 0.2 m diameter pipe is located at 0.5 m depth.

Table 3
Sensitivity analysis of how inlet parameters and pipe configuration affect the condensation rate

Inlet parameter	+20%	–20%
Temperature	2.84 kg/m/d (+58%)	0.97 kg/m/d (–46%)
Relative humidity	2.27 kg/m/d (+26%)	1.31 kg/m/d (–27%)
Velocity	2.02 kg/m/d (+12%)	1.58 kg/m/d (–12%)
Static pressure	1.98 kg/m/d (+10%)	1.66 kg/m/d (–8%)
Pipe depth	1.98 kg/m/d (+10%)	1.69 kg/m/d (–6%)
Pipe diameter	1.84 kg/m/d (+2%)	1.78 kg/m/d (–1%)

The mass movements in the simulations were restricted to considering the diffusion and bulk fluid motions caused by density and pressure gradients. Hence, mass balances were written as:

Mass conservation of air:

$$\frac{\partial}{\partial t} (S_g \theta \rho_a) = -\nabla \cdot (\mathbf{i}_a + \rho_a \mathbf{v}_g) \quad (6)$$

Mass conservation of water, independent of its phase:

$$\frac{\partial}{\partial t} (S_w \theta \rho_w) + \frac{\partial}{\partial t} (S_g \theta \rho_v) = -\nabla \cdot (\rho_w \mathbf{v}_w) - \nabla \cdot (\mathbf{i}_v + \rho_v \mathbf{v}_g) \quad (7)$$

where θ represents the porosity, S_g the gas saturation and S_w the liquid water saturation,

and ρ_a represents the air density, ρ_v the vapour density, and ρ_w the liquid water density. The air and vapour diffusive flux density vectors were denoted $\mathbf{i}_a$ and $\mathbf{i}_v$, and the phase velocity vectors of the gas $\mathbf{v}_g$ and liquid $\mathbf{v}_w$.

By writing a combined mass balance for the liquid water and the water vapour, evaporation and condensation inside the cell can be neglected in the mass balance. The sum of all water molecules is the same, independent of their current phase.

Both the mass and energy inside the cell change continuously depending on the surrounding temperature, pressure, and density [20]. When one of these state parameters in a soil element increases to a value that is higher than in the surrounding soil, heat and mass

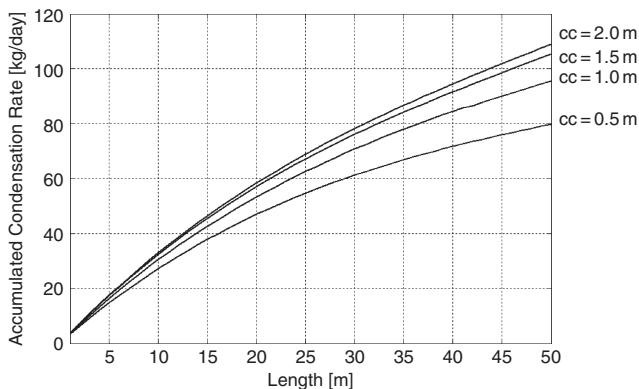


Fig. 5. Accumulated condensation rate along a pipe system with different spacing.

are transferred from that cell to regions of lower potentials.

When pressure differences arise due to heating, gravity, capillarity or mass fluxes, the mass transfer velocity is here calculated as

$$\mathbf{v}_\beta = -\frac{k_i k_{r\beta}}{\mu_\beta} (\nabla p_\beta + g \rho_\beta \nabla y) \quad (8)$$

where k_i is the intrinsic permeability, k_r is the relative permeability, μ is the viscosity, p is pressure, g is gravity, and y is the vertical coordinate axis. The index β denotes either the gas or liquid phase.

Diffusive flux densities of the air and vapour components in the gas phase are functions of the density gradients and the temperature dependent microscopic diffusion coefficient of the air-vapour mixture, D [20]. The latter is corrected for the tortuous path of the gas in the pores by the factor τ and the actual gas volume in the cell by (θS_g) , [21]:

$$\mathbf{i}_\alpha = -D\tau\theta S_g \nabla p_\alpha \quad (9)$$

where the index α denotes either dry air or water vapour.

The interaction between the diffusive and convective fluxes of the water is especially important for heat transfer. When the soil around the pipes in the CI system becomes heated from the air dehumidification process, condensed water entering the soil through the perforations may partially evaporate, and contribute to a certain cooling effect around the pipe. The diffusive vapour flux transports the vapour to cooler areas in the soil, where it condenses. In doing so, the latent heat of vaporization is released further out from the pipe, resulting in a more efficient air cooling inside the pipes.

The enthalpy change of the matter inside the considered soil cell per unit of time is written as the change in heat flux in and out

of the cell and the latent heat released or absorbed from condensation or evaporation of the water present within [22]:

$$\frac{\partial Q}{\partial t} + LE + \nabla \cdot \mathbf{q} = 0 \quad (10)$$

In the above expression, Q is the total enthalpy of all matter in the cell, L is the latent heat of vaporization, E is the mass of evaporated water inside the cell per unit time, and $\mathbf{q}$ is the total heat flux through the cell boundaries, when considering heat conduction and convection:

$$\begin{aligned} \mathbf{q} = & -k\nabla T + (T - T_0) \\ & \cdot (c_{pw}\rho_w\mathbf{v}_w + c_{pa}(\mathbf{i}_a + \rho_a\mathbf{v}_g) \\ & + c_{pv}(\mathbf{i}_v + \rho_v\mathbf{v}_g)) \end{aligned} \quad (11)$$

The thermal conductivity, k , was calculated as the weighted mean of the constituent phases present inside the soil volume, and the specific heat capacities, c_{pa} , c_{pv} , and c_{pw} , of the three non-solid components were considered constant.

In an unsaturated soil, the water pressure driving the water convective flux is a function of the capillary suction, p_c , and the gas pressure, p_g , in the soil. While the gas pressure is mainly governed by the temperature, the capillary suction depends on the water saturation in the pore space.

To analytically describe the capillarity-saturation relation of a soil is impossible because of the irregular pore geometry in the soil [21]. For this reason, many scientists have attempted to derive macroscopic models for this relation. Among the most widely used models are those described by Brooks and Corey (1964) [23] and van Genuchten (1980) [24]. In both these models parameters are fitted to the Soil Water Retention Curve (SWRC), which relates experimentally measured capillary suction to known effective water saturation degrees, S_e . The parameters

of the obtained curve shape are specific for the tested soil.

In the current model of the subsurface CI system, the theories developed by van Genuchten [24] were used to describe the capillary suction dependence of the water saturation degree:

$$p_c = \frac{-\rho_w g}{a} \cdot \left(S_e^{-1/m} + 1 \right)^{1/n} \quad (12)$$

where a and n are shape parameters to the SWRC, and $m = 1 - 1/n$. The flux densities of the liquid and gas phase in the soil are not only dependent on the pressure gradients. The effective permeability, $k_i k_{r\beta}$, determines how well each phase can be transported, and is a result of the shape and size of the pores, tortuosity, and effective water saturation. While the intrinsic permeability is a material constant, the relative permeability varies from 0, when no transport is possible, to 1, when optimum transport can be achieved at a given pressure gradient through the soil cell [21].

The van Genuchten model in conjunction with the theories developed by Mualem (1976) [25] on how to predict hydraulic conductivity in unsaturated soils yields the relative permeability of air and water in the vadose zone of the soil:

$$k_{rw} = S_e^{1/2} \cdot \left[1 - \left(1 - S_e^{1/m} \right)^m \right]^2 \quad (13)$$

$$k_{ra} = (1 - S_e)^{1/3} \cdot \left(1 - S_e^{1/m} \right)^{2m} \quad (14)$$

During the development of the subsurface CI system simulation model, several delimitations and simplifications were made in the theory. The main assumptions were:

3.2.1. Drainage pipe

- The inlet airflow properties were constant during the daily irrigation, since the air

humidification process inside the solar stills was omitted.

- The airflow inside the pipe was considered frictionless and the humid air was assumed to behave as an ideal gas to simplify the energy and mass conservation of the flow.
- No air was assumed to infiltrate the soil at night to reduce sharp temperature variations near the pipe wall that would influence the convergence of the model.

3.2.2. Soil matrix

- The soil was considered homogeneous in structure and composition so that the thermal conductivity of the particles, intrinsic permeability and porosity could be set to constant values throughout the soil matrix.
- No swelling of the soil volume occurred.
- The dissolution of air in water, surface diffusion, and the effects of hysteresis for relative permeability and capillary pressure were neglected.
- The soil was considered free from salts and organic materials; hence, osmotic and chemical reactions could be neglected.
- To obtain a more rapidly converging model, the intrinsic permeability was set lower than usually in sandy soils.
- The residual water saturation was set to 0.1, indicating that 10% of the pore space consisted of adhesive water. At water saturations below this limit, also known as the permanent wilting point of plants, capillary suction increases rapidly. A water saturation of 0.1 indicates effective water saturation, S_e of zero.

3.2.3. Boundaries

- Although the gas residual saturation was assumed to be zero in the soil, it was given a lower limit of 0.09 around the pipe perimeter to allow some humid air to enter

the soil. This is because in reality, the gas pressure from the airflow in the pipes enables air to infiltrate the soil through the build-up of preferential paths through the pore network.

- Condensed water was assumed to spread uniformly through the pipe perforations and distribute evenly around the pipe perimeter, as well as instantly flow into the soil surrounding the pipe, regardless of the pressure and saturation. Because of the restriction in gas saturation at the soil-pipe boundaries, any condensed water resulting in effective water saturation degrees higher than 0.91 around the pipe wall was removed from the mass balance and assumed to have re-entered the pipe.
- Water transported to the ground surface during one simulation time-step was removed so that the water saturation at the surface could be constant. The amount of removed water was, however, saved and used to determine a fictive surface evaporation.
- The vapour flux to the ambient air at the ground surface was a function of the vapour density gradient from the node below the surface and the surface node. The ambient air pressure and relative humidity could therefore be excluded.

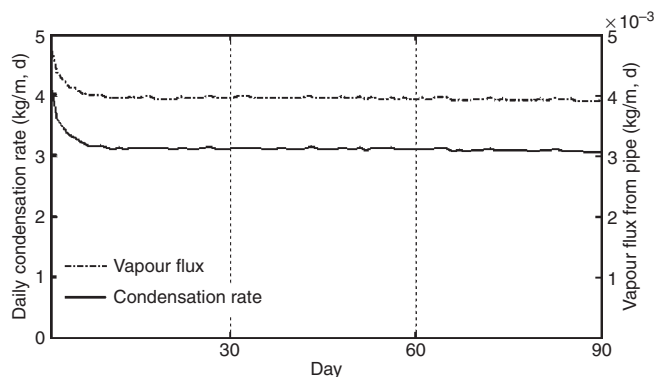
- No solar radiation or precipitation considered, though they could greatly affect the simulation results.
- The wind was considered constant, but had no effect on any of the phase fluxes below the soil surface.
- The groundwater table was at a constant temperature and depth, simplifying the boundary conditions. Water transported to the groundwater was thus removed from the model.

3.2.4. Initial conditions

- The ground temperature was initially set to the ambient mean temperature.
- The initial effective soil water saturation was 0.2.

Figure 6 shows the condensation rate inside the 50 m long drainage pipe and the vapour flux through the perforations during the first 90 days of operation. The pipe perforations in the present model contributed to an increased heat transfer from the pipe. Compared to the model for drinking water production the condensation rate was increased 72%, from 1.8 to 3.1 kg/m/d when using perforated pipes. This enhanced condensation rate is due to the mass transfer of water vapour from the soil around the pipe,

Fig. 6. Daily mean condensation rate (—), and vapour flux through perforations (---), per pipe meter during the first 90 days of irrigation.



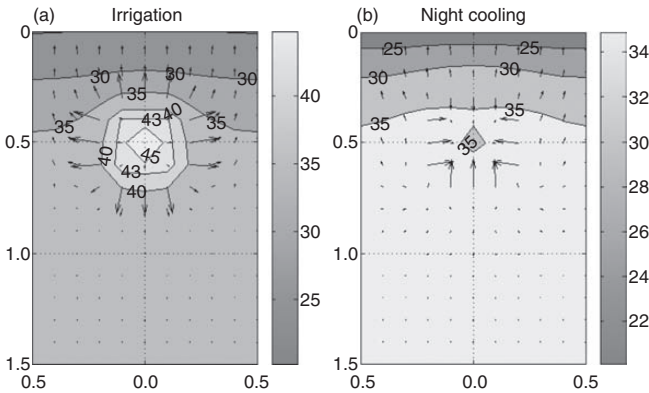


Fig. 7. Temperature and total vapour flux density in the midsection of the pipe during the 6th hour of a) irrigation and b) night cooling on the 91st day. The centre of the 0.2 m diameter drainage pipe is located at 0.5 m depth.

transporting sensible and latent heat from the vicinity of the pipe to cooler areas.

As can be seen in Fig. 7 the temperature between the pipes is below the critical rooting temperature, rendering the heating tolerable for most crops. Still, because the temperature inside the pipe is above 45°C root intrusion in the pipes are prevented, which is considered one of the most common problems in using subsurface irrigation schemes.

Figure 7 also shows the total vapour flux (diffusive and convective) in the soil, directed out from the pipe during daily irrigation and towards the pipe during the night, when cool ambient air is circulated through the pipes to restore the ground's cooling capacity.

Comparing the soil temperature in the middle cross-section of a system using non-perforated pipes (Fig. 4) to a system with drainage pipes (Fig. 7), the temperature in the soil at greater depths than the pipe is seen to be approximately 5°C higher in the system for drinking water production. In the subsurface irrigation system, the soil temperature near the pipe also changes more rapidly during the day. These differences are due to the mass transfer in the soil in the present model.

Of note from the flow directions, is that it is the diffusive flux that dominates the total flux density of vapour. A higher intrinsic permeability (leading to greater bulk fluid motions) would result in a stronger vapour flux from the pipe during the night and less vapour transport from the pipe during the day.

Since the soil-pipe system has reached a diurnal steady state on the considered day, the heat released during the day is completely removed during the nightly cooling. According to Fig. 8 the heat released per unit pipe surface area of drainage pipe ranges from 340 W/m² during the day to minus 70 W/m²

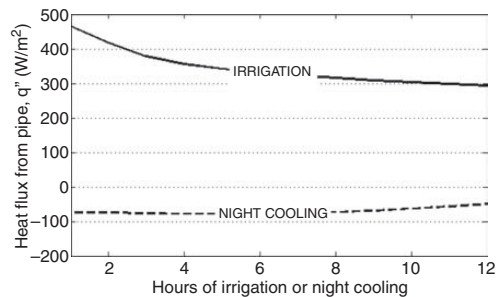


Fig. 8. Heat flux from the pipe during irrigation and nightly ambient air cooling the 91st day.

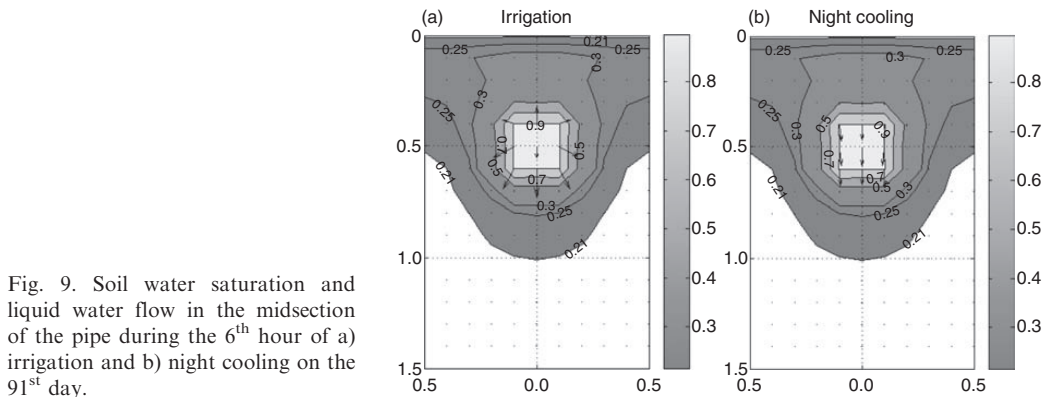


Fig. 9. Soil water saturation and liquid water flow in the midsection of the pipe during the 6th hour of a) irrigation and b) night cooling on the 91st day.

at night. The heat transported through the ground surface and to the ground water then becomes 170 W per unit ground surface area.

The heat from the drainage pipe is 55% higher than the corresponding heat release from the non-perforated pipe. This value agrees well with the measurements on underground condensation of humid air performed by Göhlman (1987), which resulted in 50% higher heat transfer from a buried drainage pipe compared to a non-perforated pipe.

In Fig. 9, the effective soil water saturation is plotted with the liquid water flux vector field. Due to the nightly cooling of the pipes and the ground surface, water accumulates above the pipe burial depth. However, including solar radiation in the calculations would heat the ground from above during the day, and reduce the upward flux.

The water movement to greater depths is mainly driven by water pressure gradients, but because the intrinsic permeability is set to a low value, the diffusive vapour flux caused by density gradients is the dominating water transport mechanism. A higher intrinsic permeability would thus cause a greater downward liquid water flux and increase the condensation rate, but would also require a more detailed simulation scheme to obtain a converging model.

Since the temperature and humidity in the soil decrease with increasing pipe spacing, wide spacing should be used with temperature sensitive crops. Planting in double rows on either side of the pipe location would result in a cooler soil environment in the rooting zone. Fig. 10 shows the mean condensation rate and irrigation yield using different pipe spacing.

Table 4
Dependence of mean condensation rate on selected inlet parameters

Inlet parameter	+20%	−20%
Temperature	5.86 kg/m/d (+90%)	1.12 kg/m/d (−63%)
Relative humidity	3.89 kg/m/d (+26%)	2.19 kg/m/d (−30%)
Velocity	3.48 kg/m/d (+11%)	2.71 kg/m/d (−13%)

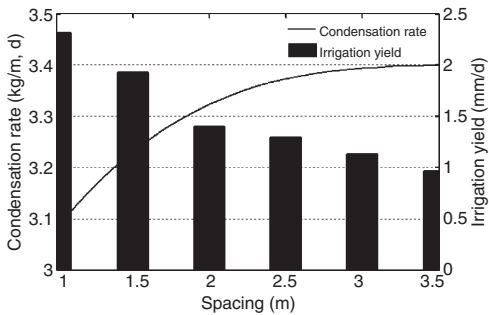


Fig. 10. Influence of pipe spacing on condensation rate (line) and mean irrigation yield (bar) during the 91st day of operation.

Compared with the drinking water production system [14], the minimum distance for a maximum condensation rate is



Fig. 11. Test rig at LTU, Luleå, Sweden. The drainage pipe is placed on a 0.75 m layer of sand in the insulated box. Another 25 cm of sand will cover the drainage pipe.

approximately 1.3 m wider in the CI system. This is explained by the improved heat transfer as a result of the mass transport in the ground, i.e. heat is spread further out from the pipe.

When varying the inlet airflow parameters in the subsurface irrigation system $\pm 20\%$, the temperature was found to have an even greater effect on the condensation rate than in the drinking water production system (Table 4). The different results are basically due to the strongly coupled relationship between the mass transfer and the temperature gradients in the present model. A higher airflow temperature also heats the surrounding soil, thereby increasing the evaporative cooling near the pipe and the mass flux of water to cooler regions. Warmer air also carries more vapour.

An increase in inlet air temperature of 20% (from 60 to 72°C) increases the condensation rate in the pipes increased by 90%, resulting in an effective irrigation rate of 4.1 mm/d. The soil was also heated by an additional 5–10°C compared to the reference soil profile in Fig. 7, which is too warm for crop roots. In a system where these high temperatures can be achieved, large pipe spacing should thus be used.

4. Field testing & experiments

To evaluate the developed theoretical models, an experimental indoor test rig was constructed at Luleå University of Technology, Sweden (Fig. 11). The setup is designed to resemble the conditions used in the numerical simulations of the subsurface CI system [26].

The soil to be irrigated is enclosed in a 3 meter long container of cross-section 1.1 m². The corrugated drainage pipe has an inner diameter of 50 mm, and the sand fraction is 0.5–1.0 mm. The vertical sides and the



Fig. 12. Pilot plant at the INRGREF in Tunisia.

bottom of the container are water tight and thermally insulated.

The sand is of homogeneous size and free from salts and organic compounds. The Soil Water Retention Curve and the thermal conductivity relationship to temperature and water saturation will be determined for the sand.

Measuring the inlet and outlet humid airflow properties, and logging the temperature distribution in the middle cross-section of the sand, provides a rough estimation of the heat and mass transport in that cross-section. To determine the transfer rates more accurately, the soil humidity must be measured as well. The test rig experiments will be conducted during the spring 2006 so that the validations of the simulation models can be carried out during the latter part of 2006.

Pilot plants on CI systems have been constructed in Tunisia and Algeria. The results from these plants will be evaluated in further research to estimate the feasibility of Condensation Irrigation. Fig. 12 shows the Tunisian pilot plant, where the subsurface irrigation system is tested. The airflow to both fields is warmed and humidified by solar collectors,

and the airflow properties, soil temperature and humidity are logged. During 2006, a second test plant will be constructed inside a greenhouse on the same area.

5. Conclusion

Two kinds of CI systems have been studied thus far, one for drinking water production and one for subsurface irrigation. Both systems were theoretically analysed through numerical simulations of the mass and heat transfer in the soil-pipe system during 90 days of operation for a presumed reference case.

The theoretical simulations of the drinking water production system resulted in a daily mean water production rate of 1.8 kg/m/d in a 50 m long pipe. Using drainage pipes for subsurface irrigation resulted in a diurnal steady state condensation rate of 3.1 kg/m/d and a mean irrigation yield of 2.3 mm/d.

When designing a CI system, the air humidification construction will play a vital role in the production efficiency, since the air inlet temperature and humidity are the

dominant factors influencing the water production and irrigation yield. Increasing the airflow temperature by 20% (from 60 to 72 °C) at the pipe inlets, the condensation rate in the non-perforated pipes increased to 2.8 kg/m/d. In the drainage pipes, this temperature increase resulted in a condensation rate of 5.9 kg/m/d and an irrigation yield of 4.1 mm/d.

Since solar radiation was excluded in the calculations, a shallow pipe depth was preferred for high water production and irrigation yields. In reality solar radiation would greatly increase the ground surface temperature, and thus lower the condensation rate in the pipes. The zone for water accumulation could also be transferred to lower depths, which would advocate greater pipe depths.

The pipe spacing will also influence the efficiency of the system. The maximum drinking water production rate was found at a pipe spacing of approximately 2.2 m in the reference system. Increasing the spacing further would not yield higher water production rates. Corresponding spacing for maximum condensation rate in the subsurface irrigation system was found at approximately 3.5 m. The wider spacing required to obtain this maximum when using drainage pipes is due to the mass transfer of water in the soil, which transports heat further out from the pipes.

When irrigation is intended, the CI system must be able to maintain both soil humidity and temperature at levels suitable for crops. According to experiments performed by Hausherr and Ruess (1993), irrigation by underground condensation of humid air halved the crop water need. Assuming this result to be generally valid, crops requiring 4.6 mm/d or less could theoretically be cultivated in the reference system.

5.1. Future work

In the continued work on developing the Condensation Irrigation system, the following will be undertaken:

Validation of simulation model by laboratory experiments: The laboratory experiments at LTU in Sweden will be used to validate and improve current simulation models.

A simulation model including solar radiation and plants: Further calculations on the subsurface CI system will be conducted where solar radiation and the leaf and root development of plants are included.

Alternative system designs: Preheating the feed water to the solar still by circulating it in separate pipes inside or around the buried drainage pipes, would enhance the airflow dehumidification rate. The potential for this preheating scheme will be theoretically examined.

System design criteria: Based upon results from the theoretical simulations, laboratory experiments, and field tests in Tunisia and Algeria, guidelines for the design of a Condensation Irrigation system will be presented and documented.

Pilot plant in Libya: A pilot plant for Condensation Irrigation will be constructed at the Al Fateh University in Libya based on the developed system design criteria.

Symbols

- a — van Genuchten parameter
- C — Accumulated condensation rate, kg/d
- c — velocity, m/s
- cc — pipe spacing, m
- c_p — specific heat capacity at constant pressure, J/kg/°C
- c_v — specific heat capacity at constant volume, J/kg/°C
- D — diffusion coefficient of air-vapour, m²/s
- D — pipe diameter, m
- d — pipe depth, m

E — evaporation rate of vapour in the soil, $\text{kg/m}^3/\text{s}$
 f — friction factor,
 h — thermal convection coefficient, $\text{W/m}^2/^\circ\text{C}$
 h_m — mass convection coefficient, m/s
 i — diffusive flux density, $\text{kg/m}^2/\text{s}$
 k_r — relative permeability, -
 k — thermal conductivity, $\text{W/m}/^\circ\text{C}$
 k_i — intrinsic permeability, m^2
 L — latent heat of vaporization, J/kg
 m — mass, kg
 m — van Genuchten parameter
 n — number of time steps, -
 n — van Genuchten parameter
 p — pressure, Pa
 Q — enthalpy, J/kg/K
 q — heat flux, W/m^2
 S — degree of saturation, m^3/m^3
 T — temperature, $^\circ\text{C}$
 t — time, s
 v — fluid phase velocity, m/s
 w — absolute air humidity, $\text{kg vapour/kg dry air}$
 y — vertical direction, -
 z — direction along the pipes, -
 ρ — density, kg/m^3
 θ — porosity
 ϕ — relative humidity
 α — thermal diffusivity, m^2/s
 τ — tortuosity
 μ — viscosity, Pa s

Subscripts

a — air
 aa — ambient air
 av — humid air
 c — capillary
 g — gas
 e — effective
 i — position in z-direction
 in — inlet
 v — vapour
 w — liquid water
 p — solid particle

p — pipe
 r — relative
 0 — reference

Superscript

· —per second

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Paper III

**Modeling Non-isothermal multiphase transport in porous
media using the modified Picard method**

Lindblom, J., Frishfelds, V., Lundström, T.S.

Submitted

Modeling Non-Isothermal Multiphase Transport in Porous Media using the Modified Picard Method

J. Lindblom^{a,*}, V. Frishfelds^b, T.S. Lundström^c

^aArchitecture and Water, Luleå University of Technology, SE-971 87 Luleå, Sweden, ^bUniversity of Liepaja, Liela iela 14, LV-3401 Liepaja, Latvia, ^cFluid and Experimental Mechanics, Luleå University of Technology, SE-971 87 Luleå, Sweden

Abstract

This paper presents a non-isothermal, two-dimensional, two-phase, finite element model, based upon the modified Picard method. The developed model CI2D is validated with isothermal experiments, and verified against a published, non-isothermal numerical example from the established simulator TOUGH2. For both isothermal and non-isothermal cases, the developed model produces robust, and accurate results, with excellent mass balance.

1 Introduction

Condensation Irrigation (CI) is a combined desalination and subsurface irrigation system that makes use of saline water for supplying clean irrigation. The system is divided into two parts, in which the first, solar stills are used for evaporating non-potable water. The formed vapour heats and humidifies ambient air that flows above the water surface inside the stills. The warm, humid air is thereafter led into the second part of the system, comprising horizontal arrays of underground drainage pipes [1, 2].

While flowing through the pipes, the air is cooled by the ground and vapour precipitates as freshwater inside the pipes. The perforations in the pipe wall enable the formed freshwater to percolate into the surrounding soil, and thereby irrigate

it. Some humid air also penetrates the perforations, which further increases irrigation by vapour condensation in the cooler ground. The airflow through the ground transports mass and heat away from the pipe and thus aerates the soil, which is important for high crop yield [1, 2]. This is a novel low-flow subsurface irrigation system, that effectively reduces water losses such as surface evaporation and deep percolation.

The subsurface irrigation described above is in theory generic and applies to other fields, such as soil remediation, as it involves warm vapour and air injections into porous media. There are a few published numerical models describing thermal multiphase flow in porous media. Among the first of these was the finite difference simulator named TOUGH (Transport Of Unsaturated Groundwater and Heat), developed by Pruess [3, 4] at the Lawrence Berkeley National Laboratories in 1987. The code was able to simulate coupled air, vapour, water and heat in porous and fractured media. TOUGH was in 1992 further expanded by Falta et al. into the simulation model STMVOC [5, 6], in which non-aqueous-phase liquids (NAPL) were included. In 1999, the well-known TOUGH2 was introduced [4, 7], based on the previous sets of models.

Forsyth presented a robust non-isothermal model in 1993, with primary variable substitution using newton iterations and both finite elements and finite volumes [8]. Helmig et al. [9] also developed an isothermal, multiphase, multicomponent simulator in 1994. This finite element model, called MUFTE (Multiphase Flow, Transport and Energy model) was later expanded by Emmert, to apply for non-isothermal situations [11], and by Class et al. to allow for primary variable substitution [12].

In 1990, Celia et al. used a mixed form of the Richards equation for an isothermal, single phase flow problem, together with a first order Taylor expansion of the specific moisture capacity term around two subsequent iterations. This so-called modified Picard linearization was shown to be perfectly mass-conservative, even when simulating sharp wetting fronts [13]. The Modified Picard method has since then been applied in several applications (e.g. [14, 15, 16]). In 1992, Celia and Binning extended this model to include both air and water phase flux [17].

In this paper the modified Picard method is further advanced by adding thermal effects into a two-dimensional, two-phase model, and including mass diffusion within the gas phase. Finite elements are used for the spatial discretization, and the coefficients to the advective fluxes are weighted by their upstream values, since only upstream weighting can ensure monotonic solutions for transient two-phase simulations [7, 6, 18]. The reason for developing this model is to supply a tool for predicting irrigation yields in different CI schemes.

To this end, the developed model CI2D was first validated by the same experiments by Touma and Vauclin [19] that were used by Celia and Binning [17], and thereafter verified using the results from numerical experiments with the simulator TOUGH2 at Lawrence Berkeley National Laboratories published by Emmert et al. [20]. In the selected verification example, hot air and vapour are injected horizontally into an initially dry sand column. This numerical example may therefore serve as a precedent, since this situation strongly resembles the physical processes in subsurface condensation irrigation.

2 Governing Equations

The developed CI2D model is a finite element two-dimensional model for predicting mass and heat transfer of air, vapour, and water through a rigid porous medium. In the formulation of the balance equation, several assumptions must be made. In the following, the main assumptions are stated accordingly.

2.1 Assumptions

Henceforth, the term “air” and “vapour” will refer to the dry air component and to the vapour component within the gas phase, respectively, and the liquid water phase is denoted “water”.

The relative humidity inside a porous medium is different from normal ambient conditions. This is due to capillarity and adhesive forces acting between the water molecules and soil particles that keep the vapour concentration in the pores very near saturation: in very moist sand, saturated to 90% with water, the relative humidity is about $\Phi = 99.9998\%$ at room temperature. By draining the sand so that only 1% of the pores are filled with water, the relative humidity is only reduced to $\Phi = 99.9260\%$. It is therefore reasonable to approximate the pore gas humidity to $\Phi \approx 100\%$ [3].

Air solubility in the water may be described by Henry’s law [21], but since the effects of this fraction of dry air is predicted to be very small, the amount is neglected. Furthermore, effects of hysteresis, organic and chemical reactions, and mechanical dispersion are neglected in this numerical model. Due to the low flux densities, local thermal equilibrium is assumed in all phases. The liquid phase is assumed to be incompressible, and the solid matrix a rigid body with constant porosity.

Lastly, the following basic relationships are identified (e.g. [18, 19]):

$$S_w = 1 - S_g, \quad (2.1)$$

$$H_\alpha = \frac{P_\alpha}{\rho_{w0}g}, \quad \alpha = g, w \quad (2.2)$$

$$H_c = H_w - H_g, \quad (2.3)$$

in which S_w and S_g are the liquid water (w) and gas (g) phase saturation, H_c , H_w and H_g are the capillary, water, and gas pressure heads, P_α (Pa) is the pressure of phase α , $\rho_{w0} = 995 \text{ kg m}^{-3}$ is a constant reference water density, and $g = 9.81 \text{ m s}^{-2}$ is the gravitational constant.

2.2 Mass and Heat transfer equations

By separating the total pressure gradient into an external pressure head H_a (m) and potential (gravitational) pressure, Darcy law is generalized to describe the flux density of phase α :

$$\mathbf{v}_\alpha = -\frac{k_{r\alpha}k_i\rho_{w0}g}{\mu_\alpha}(\nabla H_\alpha + \nabla y), \quad \alpha = g, w \quad (2.4)$$

in which y is the vertical direction, $\mathbf{v}_\alpha$ (m s^{-1}), μ_α (Pa s), ρ_α (kg m^{-3}), and p_α (Pa), are the Darcy velocity, viscosity, density, and pressure of either the gas (g) or the liquid (w) phase. The mobility of phase α is defined as $\lambda_\alpha = \frac{k_{r\alpha}}{\mu_\alpha}$ [18].

The capillary pressure is estimated from the generalized water retention function by van Genuchten [22]:

$$p_c = -\frac{1}{a} \left(S_e^{1/m} - 1 \right)^{1/n} \quad (2.5)$$

in which a and $m = 1 - \frac{1}{n}$ are soil specific shape parameters and

$$S_e = \frac{S_w - S_{rw}}{1 - S_{rw} - S_{rg}}. \quad (2.6)$$

The relative permeabilities k_{rw} and k_{rg} are derived from the theory by van Genuchten [22] in conjunction with Mualem [18, 23]:

$$k_{rw} = S_e^{1/2} \cdot \left(1 - (1 - S_e^{1/m})^m \right)^2, \quad (2.7)$$

$$k_{rg} = (1 - S_e)^{1/3} (1 - S_e^{1/m})^{2m}. \quad (2.8)$$

The gas pressure inside the pores, P_g (Pa), is the sum of the partial pressures of dry air (a), p_a (Pa), and vapour (v), p_v (Pa). The latter is approximated as the saturated vapour pressure, p_{vm} (Pa), determined through the relationship

$$p_v \approx p_{vm} = \frac{e^{(77.345 + .0057 \cdot (T + 273.15) - 7235 / (T + 273.15))}}{(T + 273.15)^{8.2}}, \quad (2.9)$$

where T (°C) is the temperature.

The partial densities of the gas components are derived from the ideal gas law, which is a good assumption for dry air. The ideal gas assumption also holds for water vapour as long as the vapour is not over-saturated:

$$\rho_\beta = \frac{M_\beta p_\beta}{R(T + 273.15)} \quad \beta = a, v.$$

The individual mass fraction of the two components air and vapour in the gas, X_β (kg kg⁻¹), is obtained from

$$X_\beta = \frac{\rho_\beta}{\rho_g} \quad \beta = a, v \quad (2.10)$$

which means that:

$$X_v + X_a = 1. \quad (2.11)$$

The gas viscosity is a function of the gas phase composition, so that [18]:

$$\mu_g = X_a \mu_a + X_v \mu_v. \quad (2.12)$$

Within the gas phase concentration gradients continuously arise due to gas pressure or temperature gradients, or as a result of condensation or evaporation. The components within the gas drift in the opposite direction of the concentration gradient according to Fick's law of diffusion. Within the pore space, the diffusion coefficient D_0 must be adjusted with a factor τ for the tortuous path of the pores [24] and by the gas content $\rho_g S_g \theta$ (m_g³m⁻³), due to the reduction of gas flow area caused by the presence of water [25]:

$$D = \tau \rho_g S_g \theta D_0.$$

The diffusion of either dry air (a) or vapour (v), $\mathbf{i}_\beta$ (kg m⁻²s⁻¹), is hence described by:

$$\mathbf{i}_\beta = -D \nabla X_\beta \quad \beta = a, v.$$

Since the corrected diffusivity D is the same for both components, and because of Eq. (2.11) it also follows that

$$\mathbf{i}_a = -\mathbf{i}_v.$$

In the presence of temperature differences, heat is transferred through the porous media. This is done primarily by water advection in both liquid and gaseous form. Air advection as well as heat diffusion in all phases also contribute to the transport of heat. Radiation heat transfer is, however, neglected.

In the CI2D model, the heat flux, $\mathbf{q}$ (W m^{-2}), through a control volume of soil under local thermal equilibrium is a result of the water and gas advection and diffusion through the pore system together with heat conduction, according to:

$$\mathbf{q} = h_a (\rho_a \mathbf{v}_g + \mathbf{i}_a) + h_v (\rho_v \mathbf{v}_g + \mathbf{i}_v) + h_w \rho_w \mathbf{v}_w - k_h \nabla T. \quad (2.13)$$

In Eq. (2.13), h_a , h_v , and h_w (J kg^{-1}), are the enthalpies for air, vapour and water, respectively, and k_h (W m^{-2}) is a local overall thermal conductivity for the porous media. The air and liquid water enthalpies are defined as:

$$h_a = c_{p,a} (T - T_{ref}),$$

$$h_w = c_{p,w} (T - T_{ref}),$$

where $c_{p,a} \approx 1020 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ and $c_{p,w} \approx 4190 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ are the specific heat capacities under constant pressure for air and water, respectively [18]. The reference temperature is $T_{ref} = 0 \text{ }^\circ\text{C}$. The enthalpy for water vapour is obtained from the liquid water enthalpy by adding to it the vapour latent heat of vaporization, L (J kg^{-1}). For temperatures $0^\circ\text{C} \leq T \leq 100^\circ\text{C}$ [26] the vapour enthalpy is:

$$h_v = h_w + L.$$

Within the given temperature interval, L varies with temperature according to:

$$L = 2503766.7 - 2444.8T. \quad (2.14)$$

The latent heat of vaporization is a major contributor to the heat transfer in unsaturated porous media. At room temperature, the energy absorbed in the vapour upon evaporation ($L \approx 2.5 \text{ MJ kg}^{-1}$) is about 600 times the energy absorbed in

liquid water when heated 1°C. When the vapour condenses, energy corresponding to the latent heat of vaporization is released to the surrounding.

The overall thermal conductivity, k_h ($\text{W m}^{-1}\text{C}^{-1}$), of the porous media is assumed to be a function of the water saturation according to [20]:

$$k_h = 1.5 \cdot \sqrt{S_w} + 1.$$

2.3 Mass and Heat Balance Equations

In many isothermal models describing water transport in unsaturated porous media, the gas phase is assumed to be stationary or at constant pressure, i.e. near atmospheric (e.g. [13, 27]). This may be a valid assumption when the gas has a free flow path to the surface; the gas then makes use of its superior mobility and quickly evens out gas pressure differences between the soil and atmosphere. Water movements are thus not hindered by the presence of air in the soil.

When the gas is arrested in soil barriers, waterfronts, etc, the gas pressure will build up and greatly hinder the water flow. Under these circumstances the transport of gas must be considered in the calculations [17, 19]. When temperature gradients are present, the transport of the gas phase must never be neglected, since one of the most important mass and heat transfer mechanisms is the vapour transport of both latent and sensible heat from warm areas to cooler.

The air component in the gas phase is transported through the porous media by diffusion within the gas and by gas phase advection. The air mass balance is thus summarized as (e.g. [17, 18]):

$$\frac{\partial}{\partial t} (S_g \theta \rho_a) + \nabla \cdot (\mathbf{i}_a + \rho_a \mathbf{v}_g) = f_a \quad (2.15)$$

in which the term f_a ($\text{kg s}^{-1}\text{m}^{-3}$) contains any sources or sinks within the control volume, and $S_g \theta \rho_a$ ($\text{kg s}^{-1}\text{m}^{-3}$) equals the mass of air per unit volume. Making use of Eq. (2.1) and assuming the porosity to be constant in time, the time derivative of Eq. (2.15) is expanded to

$$\frac{\partial}{\partial t} (S_g \theta \rho_a) = \theta \left((1 - S_w) \frac{\partial \rho_a}{\partial t} - \rho_a \frac{\partial S_w}{\partial t} \right),$$

where θ (m^3m^{-3}) is the porosity. The dependence of air density on time can be divided into a gas pressure related and a temperature related term:

$$\frac{\partial \rho_a}{\partial t} = \frac{\partial \rho_a}{\partial p_a} \frac{\partial p_g}{\partial t} + \left(\frac{\partial \rho_a}{\partial T} - \frac{\partial \rho_a}{\partial p_a} \frac{\partial p_v}{\partial T} \right) \frac{\partial T}{\partial t}.$$

Inserting Eq. (2.2) into this expression and designating

$$\begin{aligned} a_H &= \theta(1 - S_w) \frac{\partial \rho_a}{\partial p_a} \rho_w g, \\ a_S &= -\theta \rho_a, \\ a_T &= \theta(1 - S_w) \left(\frac{\partial \rho_a}{\partial T} - \frac{\partial \rho_a}{\partial p_a} \frac{\partial p_v}{\partial T} \right) \end{aligned}$$

results in a final expression for the air mass balance according to:

$$a_H \frac{\partial H_g}{\partial t} + a_S \frac{\partial S_w}{\partial t} + a_T \frac{\partial T}{\partial t} + \nabla \bullet (\mathbf{i}_a + \rho_a \mathbf{v}_g) = f_a. \quad (2.16)$$

In unsaturated porous media the transport of water takes place in both the gas and liquid phase. While liquid water flow is generally governed by capillary action (i.e. water saturation differences), the gas transport is strongly temperature dependent. In Eq. (2.9), the exponential dependence of vapour pressure on temperature is clearly visible.

When the temperature in the soil rises near a heat source, liquid water starts to evaporate, resulting in an increase of vapour pressure. As a result, vapour moves towards cooler regions, where it precipitates as liquid water. The allocation of liquid water from the warm to the cold region causes liquid water to be drawn by capillary forces from the cool area towards the drier area around the heat source. This transport mechanism, known as the heat pipe effect, may often be the dominant means by which heat is transferred in a porous media [28].

The expression for the water balance includes both the liquid and the vapour phase, so that any phase transition of water does not influence the total mass balance of water. The mass balance equation of water is written in the following way:

$$\frac{\partial}{\partial t} (S_w \theta \rho_w) + \frac{\partial}{\partial t} (S_g \theta \rho_v) + \nabla \bullet (\mathbf{i}_v + \rho_v \mathbf{v}_g) + \nabla \bullet (\rho_w \mathbf{v}_w) = f_w$$

in which, f_w ($\text{kg s}^{-1} \text{m}^{-3}$) represents any water or vapour source or sink, $S_w \theta \rho_w$ ($\text{kg s}^{-1} \text{m}^{-3}$) and $S_g \theta \rho_v$ ($\text{kg s}^{-1} \text{m}^{-3}$) are the mass of liquid and vapour per unit volume. By the assumptions previously stated, the time derivative is expanded to:

$$v_S \frac{\partial S_w}{\partial t} + v_T \frac{\partial T}{\partial t} + \nabla \bullet (\mathbf{i}_v + \rho_v \mathbf{v}_g) + \nabla \bullet (\rho_w \mathbf{v}_w) = f_w \quad (2.17)$$

in which

$$\begin{aligned} v_S &= \theta (\rho_w - \rho_v), \\ v_T &= \theta \left(S_w \frac{\partial \rho_w}{\partial T} + (1 - S_w) \frac{\partial \rho_v}{\partial T} \right). \end{aligned}$$

The transfer of heat through the porous media, $\mathbf{q}$ (W m^{-2}), occurs through heat conduction together with gas and liquid transport, but is greatly enhanced by the latent heat released or absorbed through water phase transitions. The energy balance for a control volume of a porous media is expressed as:

$$\frac{\partial Q}{\partial t} + \nabla \bullet \mathbf{q} = f_e$$

in which f_e (W m^{-3}) represents any heat source or sink, and Q (J m^{-3}) is the internal energy of the components comprising the control volume, according to:

$$Q = \theta [u_w S_w \rho_w + (1 - S_w) (u_v \rho_v + u_a \rho_a)] + (1 - \theta) u_p \rho_p.$$

In this equation, u_a , u_v , u_w , and u_p (J kg^{-1}), are the internal energy for air, vapour, water, and solid particles. ρ_p (kg m^{-3}) is the density of the solid. The heat flux $\mathbf{q}$ (W m^{-2}) is in its turn calculated from Eq. (2.13).

By expanding the time derivative of Q and inserting Eqs. (2.1-2.2), the energy balance is written:

$$e_H \frac{\partial H_g}{\partial t} + e_S \frac{\partial S_w}{\partial t} + e_T \frac{\partial T}{\partial t} + \nabla \bullet \mathbf{q} = f_e, \quad (2.18)$$

where

$$\begin{aligned} e_H &= h_a a_H, \\ e_S &= \theta (\rho_w h_w - \rho_v h_v - \rho_a h_a), \\ e_T &= \theta (1 - S_w) \left(\rho_v \frac{\partial h_v}{\partial T} + h_v \frac{\partial \rho_v}{\partial T} + \rho_a \frac{\partial h_a}{\partial T} + h_a \frac{\partial \rho_a}{\partial T} \right) \\ &\quad + \theta S_w \left(\rho_w \frac{\partial h_w}{\partial T} + h_w \frac{\partial \rho_w}{\partial T} \right) + (1 - \theta) \rho_p \frac{\partial h_p}{\partial T}. \end{aligned}$$

3 Numerical Approximation

The strong non-linearities in several of the secondary variables greatly complicates the numerical approximations of the above balance equations. The problem must

hence be solved using implicit iterations. Newton or Picard methods are usually used for this purpose, together with a finite difference or finite element spatial discretization.

While Newton iterations have a faster convergence rate (fewer iterations), Picard iterations produce more stable solutions. Newton iterations also require much more computation effort for the calculation of the numerical Jacobian, implying that each iteration will take a longer time [29, 30].

Regardless of type of iteration method and spatial discretization used, the accuracy of the numerical solution to many multiphase flow problems depends on the selection of primary variables. When basing the water balance on saturation, the model may produce a perfect mass balance, but will only be valid in the unsaturated zone and for homogeneous media. Using water pressure as a primary variable may result in very large mass balance errors, unless the specific moisture capacity term, $C_w = \frac{\partial S_w}{\partial H_c}$, is treated correctly [13, 31, 32, 33].

Celia et al. [13] developed a mass conservative method that also were applicable in the saturated zone by stating the balance equations in a mixed form and using a so called modified Picard linearization of the temporal derivative.

3.1 Modified Picard Method

The modified Picard method is based on an implicit Euler approximation of the temporal terms in the balance equations together with a variable substitution of the water saturation variable with water pressure. By including an active air phase Celia and Binning further developed the method for two-phase flow in isothermal porous media.

Presently, the modified Picard method will be used for simulating non-isothermal situations with a two-component gas phase. To express the balance equations on the modified form, an implicit Euler approximation is firstly applied to the air balance equation. In the following equations, $k_w = k_i \lambda_w \rho_{w0} g$ and $k_g = k_i \lambda_g \rho_{w0} g$, and the indexes $t+1$ and $n+1$ denote the current time and iteration values. By adding and subtracting the former iteration value (index $t+1, n$) of the primary variables, the balance equations take the form:

$$\begin{aligned}
& a_H^{t+1,n} \frac{\delta H_g}{\Delta t} + a_S^{t+1,n} \frac{\delta S_w}{\Delta t} + a_T^{t+1,n} \frac{\delta T}{\Delta t} - \nabla \bullet (\rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) \\
& = f_a^{t+1,n} - a_H^{t+1,n} \frac{H_g^{t+1,n} - H_g^t}{\Delta t} - a_S^{t+1,n} \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - a_T^{t+1,n} \frac{T^{t+1,n} - T^t}{\Delta t} \\
& \quad + \nabla \bullet \mathbf{i}_a^{t+1,n} + \nabla \bullet \left(\rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_{w0}} \nabla y \right) \right) \\
& \hspace{20em} = R_A \quad (3.1)
\end{aligned}$$

in which

$$\begin{aligned}
\delta H_g &= H_g^{t+1,n+1} - H_g^{t+1,n}, \\
\delta S_w &= S_w^{t+1,n+1} - S_w^{t+1,n}, \\
\delta T &= T^{t+1,n+1} - T^{t+1,n}.
\end{aligned}$$

The residual of the air balance equation, R_A , is defined as the deviation of the former iteration value from the target value of the change in air mass over the time step. As the iterations progress, R_A , δH_g , δS_w , and δT approach zero, and the solution thus converges.

To complete the temporal discretization of the air balance equation, the primary variable S_w is substituted by the water pressure head, H_w , through the relationship in Eq. (2.3), and by approximating the specific moisture capacity with a first order Taylor expansion around the current and last saturation value:

$$C_w \approx \frac{S_w^{t+1,n+1} - S_w^{t+1,n}}{H_c^{t+1,n+1} - H_c^{t+1,n}} = \frac{\delta S_w}{\delta H_w - \delta H_g}$$

which is re-written as

$$\delta S_w = C_w (\delta H_w - \delta H_g). \quad (3.2)$$

Inserting Eq. (3.2) into Eq. (3.1) yields the final air balance equation:

$$\begin{aligned}
& a_H^{t+1,n} \frac{\delta H_g}{\Delta t} + a_S^{t+1,n} \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + a_T^{t+1,n} \frac{\delta T}{\Delta t} - \nabla \bullet (\rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) \\
& = f_a^{t+1,n} - a_H^{t+1,n} \frac{H_g^{t+1,n} - H_g^t}{\Delta t} - a_S^{t+1,n} \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - a_T^{t+1,n} \frac{T^{t+1,n} - T^t}{\Delta t} \\
& \quad + \nabla \bullet \mathbf{i}_a^{t+1,n} + \nabla \bullet \left(\rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_{w0}} \nabla y \right) \right) \\
& = R_A \quad (3.3)
\end{aligned}$$

By following the same procedure for the water and heat balance equation, these take the form:

$$\begin{aligned}
& v_S^{t+1,n} \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + v_T^{t+1,n} \frac{\delta T}{\Delta t} \\
& - \nabla \bullet (\rho_w^{t+1,n} k_w^{t+1,n} \nabla \delta H_w) - \nabla \bullet (\rho_v^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) \\
& = f_v^{t+1,n} - v_S^{t+1,n} \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - v_T^{t+1,n} \frac{T^{t+1,n} - T^t}{\Delta t} \\
& + \nabla \bullet \mathbf{i}_v^{t+1,n} + \nabla \bullet \left(\rho_v^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_{w0}} \nabla y \right) \right) \\
& + \nabla \bullet \left(\rho_w^{t+1,n} k_w^{t+1,n} \left(\nabla H_w^{t+1,n} + \frac{\rho_w^{t+1,n}}{\rho_{w0}} \nabla y \right) \right) \\
& = R_V \quad (3.4)
\end{aligned}$$

and

$$\begin{aligned}
& e_H^{t+1,n} \frac{\delta H_g}{\Delta t} + e_S^{t+1,n} \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + e_T^{t+1,n} \frac{\delta T}{\Delta t} \\
& - \nabla \bullet \left(h_a^{t+1,n} \rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g \right) - \nabla \bullet \left(h_v^{t+1,n} \rho_v^{t+1,n} k_g^{t+1,n} \nabla \delta H_g \right) \\
& - \nabla \bullet \left(h_w^{t+1,n} \rho_w^{t+1,n} k_w^{t+1,n} \nabla \delta H_w \right) - \nabla \bullet \left(k_h^{t+1,n} \nabla \delta T \right) \\
& = f_e^{t+1,n} - e_H^{t+1,n} \frac{H_g^{t+1,n} - H_g^t}{\Delta t} - e_S^{t+1,n} \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - e_T^{t+1,n} \frac{T^{t+1,n} - T^t}{\Delta t} \\
& + \nabla \bullet \left(h_a^{t+1,n} \left(\mathbf{i}_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_{w0}} \nabla y \right) \right) \right) \\
& + \nabla \bullet \left(h_v^{t+1,n} \left(\mathbf{i}_v^{t+1,n} + \rho_v^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_{w0}} \nabla y \right) \right) \right) \\
& + \nabla \bullet \left(h_w^{t+1,n} \rho_w^{t+1,n} k_w^{t+1,n} \left(\nabla H_w^{t+1,n} + \frac{\rho_w^{t+1,n}}{\rho_{w0}} \nabla y \right) \right) + \nabla \bullet \left(k_h^{t+1,n} \nabla T^{t+1,n} \right) \\
& = R_E \quad (3.5)
\end{aligned}$$

The three balance equations (3.3-3.5) are solved simultaneously for δH_g , δH_w , and δT , respectively, making the primary variables H_g , H_w , and T .

Since consistent time matrices frequently give rise to oscillations in the numerical solution, the consistent matrix is lumped by placing the sum of each row in the main diagonal of the time matrix [33, 31].

3.2 Spatial approximation

In the CI2D model, rectangular Galerkin finite elements are used as spatial discretization of Eqs. (3.3-3.5). The coefficients belonging to the advective fluxes (i.e. phase mobility, density and enthalpy) all assume upstream values since it is the only way to ensure monotonic solutions [7, 12, 18, 27, 30]. Furthermore, if transient two-phase flow is modeled, the advancement of the infiltration front might be calculated incorrectly unless the relative permeabilities are weighted upstream [7, 34]. It is also the experience with the current simulation model that mean values (arithmetic as well as geometric) produce non-monotone solutions when including thermal effects, where upwind values yield monotone.

4 Examples and Results

There are only few measurements or experimental examples published for thermal multiphase transport problems in porous media that can be used for validating

purposes. The validation of the CI2D model is therefore split into two parts: First the modified Picard method is validated isothermally using the same experiments by Touma and Vauclin [19] as Celia and Binning used to validate their model [17]. In a second example, a simulation of thermal multiphase flow is demonstrated by repeating one of the numerical experiments performed by Emmert et al. on horizontal hot steam and air injection into an initially dry sand [20]. The latter is rather a verification than a validation but is relevant since the program TOUGH2, that was used by Emmert et al. is well established and has been validated for a number of cases [4, 7].

4.1 Example 1: Isothermal two-phase flow

In 1986, Touma and Vauclin performed laboratory experiments on vertical water infiltration into an initially dry sand [19]. The purpose was to study the effects of air phase flow in the sand during water infiltration into a sand column with either open or sealed bottom. The results of these experiments were used by Celia and Binning to validate their proposed isothermal two-phase model using the modified Picard formulation [17].

For the numerical approximation of the performed experiments Touma and Vauclin used the theory developed by van Genuchten [22] to describe the water retention curve (Eq. 2.5), while the conductivity functions of air and water were determined through experimental observations:

$$K_w = A_w (\theta S_w)^{B_w} ,$$

$$K_a = K_{as} \frac{A_a}{A_a + h_c^{B_a}}$$

in which K_w and K_a are the absolute conductivity of the water and air phase. The parameters used for describing water content and conductivities are summarized in Table 1.

Table 1: Parameters used for water retention and conductivity functions [19]

α	0.44	mm^{-1}
n	2.2	-
θS_{ws}	0.312	$\text{mm}^3 \text{mm}^{-3}$
θS_{wr}	0.0265	$\text{mm}^3 \text{mm}^{-3}$
A_w	0.0504	mm s^{-1}
B_w	6.07	-
K_{as}	7.7778	mm s^{-1}
A_a	$3.86 \cdot 10^{-5}$	-
B_a	-2.4	-

In the second validation example published by Celia and Binning, water infiltrates into a vertical sand column with the bottom sealed for both air and water flux. Initially, the air phase is assumed to be uniform at atmospheric pressure, while the water phase pressure is a function of the gravitational field. Initial water saturation follows the resulting capillary pressure curve.

As the experiment commences water is dripped onto the surface of the sand column at a rate corresponding to a constant flux of $23.056 \cdot 10^{-3} \text{ mm s}^{-1}$. The low water flux at the inlet boundary is not enough to saturate the sand surface with water, thus allowing air to escape upward and through the soil surface.

As the water pushes downward, the air below the front is compressed to make room for the water. The high air phase mobility in the dry area below the infiltration front causes the air pressure gradient to be nearly vertical. In the wetter upper part of the sand the air flux is retarded by a decreasing air conductivity.

The air compression below the water front continues until the air pressure difference to the ambient is enough for the air to escape to the surface. Since the wet area behind the front increases during infiltration, the air pressure below the front must increase to maintain a pressure gradient large enough to transport the air to the surface.

In Figure 4.1 the advancement of the pressure profiles of the two active phases are displayed for every passing ten minutes, as calculated with the current theory and as presented by Celia and Binning [17]. Above the water front the air pressure gradient directs the air upward towards the surface, while the air is compressed downward below the water front.

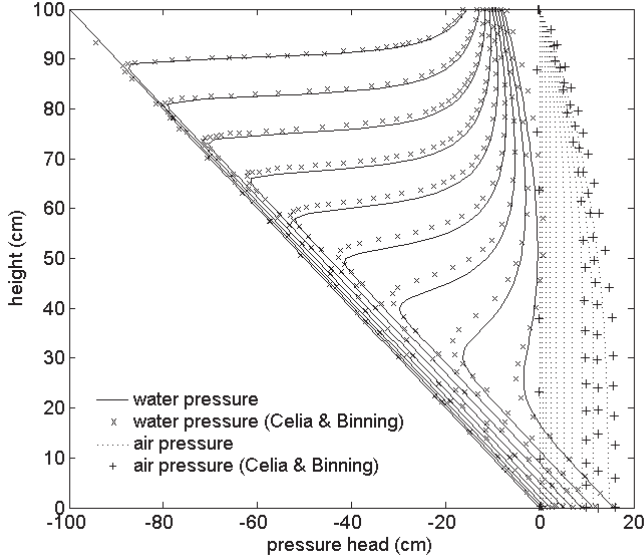


Figure 4.1: Pressure profiles for the water and air phase in the sand column after 0, 10, ..., 90 minutes

The mass balance of both water and air phases are less than $3 \cdot 10^{-4}$, so the isothermal CI2D model based on the work of Celia and Binning can be said to be nearly mass conservative.

4.2 Example 2: Non-isothermal two-phase flow

Emmert et al. used the simulator TOUGH2/T2VOC at Lawrence Berkeley Laboratory for predicting two-dimensional flow patterns during hot steam and air injection into a sand column [20]. The simulated sand profile was 0.74 by 1.35 m, with a thickness of 0.10 m. The geometry had three inlets and three outlets, symmetrically located on the left and right vertical boundaries, respectively. Initially, the temperature was set to 10°C everywhere and the gas phase pressure was at atmospheric levels, i.e. $P_g = 101330$ Pa. The sand was assumed to be at residual saturation ($S_{rw} = 0.20$), and the initial water pressure head was calculated using the corresponding capillary pressure and gas phase pressure.

In the simulated example, constant vapour and air fluxes of 4 g s^{-1} and 2 g s^{-1} at 100°C were applied to each inlet, while the primary variables, T , S_{rw} , and P_g , were assumed constant at the outlet. A van Genuchten relationship, Eq. (2.5)

was assumed to describe the water capillarity-saturation relationship. The relative permeability function in Eq. (2.7) was used for the liquid, while the gas relative permeability was described by [7]:

$$k_{rg} = (1 - S_e)^2 (1 - S_e^2).$$

Relevant parameters are displayed in Table 2.

Table 2: Parameters used in Example 2 [20]

a	0.001614	Pa^{-1}
m	1.8416	-
k_i	$5 \cdot 10^{-10}$	m s^{-1}
θ	0.30	$\text{m}^3 \text{m}^{-3}$
τ	0.62	
ρ_p	2243	kg m^{-3}
S_{rw}	0.2	$\text{m}^3 \text{m}^{-3}$
S_{rg}	0.2	$\text{m}^3 \text{m}^{-3}$

The liquid water saturation profile after 270 s is shown in Figure 4.2 for simulations using TOUGH2 and CI2D. The liquid saturation is in both models almost vertical in shape, except in the wettest area, where the relative water permeability is high enough to allow water to be pulled downward by gravity.

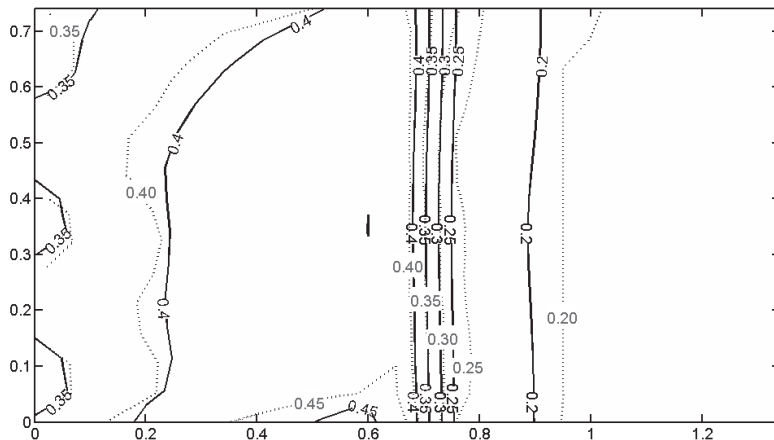


Figure 4.2: Water saturation profile after 270 seconds from simulations using TOUGH2 [20] (dotted) and CI2D (line).

The saturation profile from simulations using TOUGH2 and CI2D in Figure 4.2 varies slightly, as the CI2D profile seems a bit drier both before, after and inside the wet region. Since both geometries contain the same amount of water, these variations should depend on the post-processing of the nodal values.

As 100°C air and vapour is injected into the initially cool sand, the sand just outside the inlets is quickly heated by the latent heat release from the steam condensation. The increasing temperature in the condensation zone drives the vapour further and further away from the inlet before it condenses. This results in a nearly horizontal temperature profile up to the condensation front, where the temperature drops rapidly.

Before the front, heat is transferred primarily by gas advection, but in the absence of latent heat release from condensation, the temperature increase is marginal. The resulting temperature profiles after 270 s simulation is shown in Figure 4.3 for both simulators.

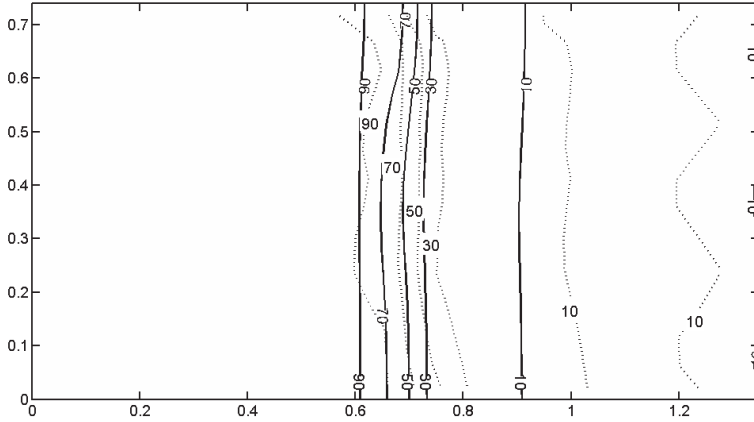


Figure 4.3: Temperature contour lines after 270 s. Results from TOUGH2 is marked by dotted lines [20]. Results from CI2D are marked by solid lines.

During the simulation, the added heat and mass of vapour and dry air either accumulated in the sand or exited the geometry through the outlets. In Table 3 the respective amounts after 270 s simulation using CI2D are presented, together with the resulting balance:

Table 3: Balance of water (all phases included), air, and energy after 270 s in CI2D.

	Water (kg)	Air (kg)	Energy (J)
Injection through inlets	3.2400	1.6200	8 801 946
Accumulation inside geometry	3.2278	-0.0131	8 754 162
Lost through outlets	0.0124	1.6330	43 252
Balance	$6.2 \cdot 10^{-5}$	$6.2 \cdot 10^{-5}$	$5.1 \cdot 10^{-4}$

Since pore gas has been forced to make room for the increasing water mass, less air resides within the geometry at the end of the simulation, and the air accumulation term is for this reason negative. The mass fraction of water (in all phases) lost through the outlets is exactly equal to the vapour mass fraction of 10°C vapour saturated air at atmospheric pressure (i.e. $X_v = 0.76\%$).

By including an air flux source at the inlet, the gas phase gradient through the geometry is increased. The vapour is then transported further along the geometry before condensation occurs. This was clearly demonstrated by the theoretical study by Emmert et al. [20], where the water displacement in the present simulation model was compared to the case when only hot vapour was injected into the geometry. Without the non-condensable air component in the gas phase, the gas advection, which stands for the majority of the heat transfer, is not able to carry the vapour across the forming condensation front, and liquid water thus accumulates just outside the inlets.

Since the air component in the gas flux plays such an important role in the mass and heat transfer through a porous media, the air cannot be assumed to be immobile when modeling thermal multiphase flow.

5 Conclusion

A thermal two-phase model was developed for obtaining theoretical predictions of irrigation yield in condensation irrigation (CI) systems. In a CI system, drainage pipes are buried horizontally in the ground and used for conducting warm, humid air. While flowing through the drainage pipe, vapour precipitates on the inner walls of the pipes, and percolates into the ground as irrigation water. Some of the humid air also penetrate the perforations, and vapour condenses in the cooler ground, thereby irrigating plants directly in the root zone.

The current two-dimensional model CI2D was first validated by reproducing isothermal experiments performed by Touma and Vauclin in which air and water pressures were measured during vertical infiltration.

One verification example with thermal effects was then selected from the numerical experiments performed by Emmert et al. with the simulation program TOUGH2/T2VOC. In the example hot air and vapour are injected horizontally into a thin sand box, and vapour is transported to cooler regions, where it condenses. This particular verification example was well suited for verifying the CI2D program, due to the similarity to the future CI simulations. The simulation results from CI2D were in agreement with both examples, and mass and energy balances were well within acceptable levels. It was therefore concluded that the simulator CI2D should be able to produce accurate estimations of a CI system.

From numerical experiments, it was evident that upwind values of gas and liquid phase mobilities are necessary to ensure monotonic, stable solutions in these types of problems. From the verification example, the significance of an active gas phase was also observed, indicating that the gas phase should always be included in water transport models with thermal effects.

In the future the current model will be expanded with algorithms for estimating humid airflow through a perforated pipe, with mass and heat transfer couples to the soil matrix. Plant root suction, and ground surface heat and mass transfer will also be included.

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Paper IV

Condensation Irrigation - Operation simulation and feasibility study

Lindblom, J., Nordell, B.

Submitted

Condensation Irrigation - Operation Simulation and Feasibility Study

J. Lindblom* and B. Nordell

Dept of Civil, Environmental and Natural resources engineering, Luleå
University of Technology, SE-971 87 Luleå, Sweden

Abstract

Condensation irrigation (CI) is a novel irrigation technology that combines solar desalination with subsurface irrigation. Ambient air is warmed and humidified in solar stills and then led into arrays of horizontally buried drainage pipes. While flowing through the pipes, the air is cooled by the underground and vapor precipitates as distilled water. The formed water and some humid air infiltrate the soil through the pipe perforations, supplying irrigation and soil aeration. To investigate the feasibility of this type of system, a reference scenario was analyzed using the validated simulator CI2D. The potential irrigation yield was estimated to 3.44 mm d^{-1} in the reference case, of which 19.8% of the water was taken up by plant roots. The temperature in the root zone was not deemed too high. Alternative CI solutions were also investigated, in which the crop water uptake reached 48% of the supplied water.

1 Introduction

Irrigation water shortages frequently cause crop yield reductions in the semi-arid and arid regions. To alleviate the water scarcity, marginal quality water is sometimes used for irrigation (Beltran (1999); Qadir et al. (2007); Ragab et al. (2005)). Since this strategy involves risk of degrading land and nearby water sources, the water should be purified prior to irrigation whenever possible (Tedeschi and Menenti (2002)).

Solar desalination is a distillation technique where saline water is evaporated by solar thermal energy and thereby separated from the salt. When the vapor later condenses, freshwater is formed. Solar stills are passive, low-tech, low-cost devices, which makes them suitable for rural areas (Fath (1998)).

Condensation Irrigation (CI) is a novel technology in which solar stills are connected to a subsurface irrigation system. Ambient air inside the solar stills is heated and humidified by the evaporating saline water. This warm, humid air exits the stills into buried drainage pipes where it is cooled and the vapor precipitates as liquid freshwater along the pipes (Fig. 1.1). The perforations in the pipe walls enable the formed water to infiltrate the surrounding soil, together with some of the humid air. The soil is in this manner irrigated and aerated close to the rooting zone (Lindblom and Nordell (2007)). If non-perforated pipes are used in the ground, the condensed water from the pipes can be collected at the pipe endings and used for drinking or other purposes (Lindblom and Nordell (2006)).



Figure 1.1: Plan showing buried perforated pipes in the condensation irrigation system. In the layout used as reference in this study, there is one crop row planted in-between each pipe.

During the daily irrigation the soil is gradually heated by the mass and heat transfer from the pipes, which reduces the condensation rate in the pipes and increases surface evaporation losses. To reduce this ground heating, cold ambient air is circulated through the pipe system at night. When irrigation is not needed, the soil temperature can be further reduced by nightly air cooling (Fig 1.2).

The irrigation yield of the CI system depends on parameters such as pipe configuration, climate, soil type, and expected inlet humid airflow properties (Kandelous and Simunek (2010); Lindblom (2006)). For example, although requiring larger land areas, using wide spacing between the pipes results in higher irrigation rate per pipe and lower installation costs. This will also lead to reduced surface evaporation rates due to the lower average soil humidity and temperature. Pipe lengths affect the average irrigation yield, since the condensation rate decreases along the pipe due to the cooling airflow.

Water demand, rooting depth, and temperature sensitivity of the crop must also be taken into account. In general, temperatures up to 32°C are considered stimulating for root growth (Arai-Sanoh et al. (2010)), while higher tem-

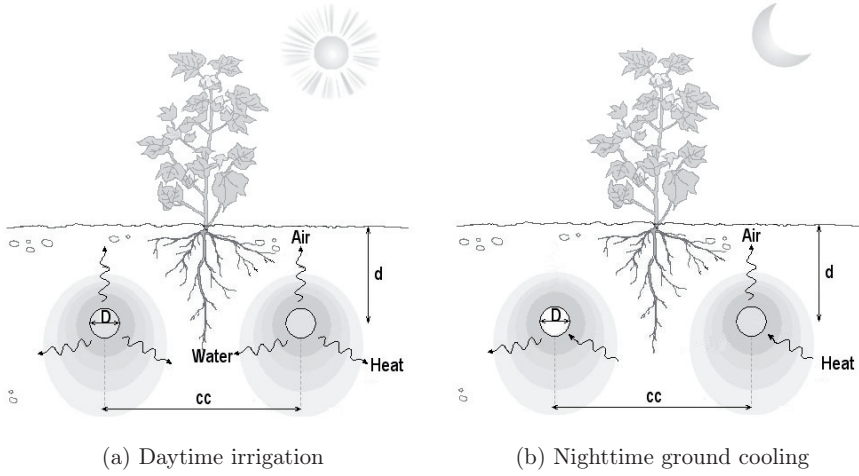


Figure 1.2: Section showing irrigation pipes and crop with the principle mass and heat flux modes. The pipe spacing, depth, and diameter are denoted in the figure

peratures can reduce root development and respiration (Rachmilevitch et al. (2006)). In the CI system, the pipe configuration should therefore be designed so that this critical temperature is reached at the pipe walls, but never in the soil between two pipes. In so doing, plant roots are free to develop in-between the pipes, but not into the pipes, where they would block the airflow. Root intrusion is a common problem in subsurface irrigation systems, and is usually solved by injecting herbicides through the pipes (Camp (1998)). In the CI system, this is hence avoided naturally by the high temperature of the pipe airflow .

Other advantages gained from subsurface irrigation is that water use becomes more efficient, surface losses from evaporation and run-off is reduced (Camp (1998)), harvesting becomes easier, and surface crusts are prevented. CI has the additional advantage of being able to safely use saline or otherwise polluted water as a source for irrigation. When using solar thermal energy for the air humidification, the irrigation rate then also follows the irrigation need: during sunny days, more water is produced in the subsurface pipes than on cloudy or cold days.

Hausherr and Ruess (1993) performed field experiments on one type of CI system in 1985-89. Their results showed that tomatoes required only half of the irrigation water (300 mm) compared to conventionally irrigated tomatoes. Other experiments on CI have been carried out at the National Research Institute for Agricultural Engineering in Tunisia (Lindblom (2006)), and a small

scale CI has been tested at LTU (Lindblom and Nordell (2011)).

The principle of the CI system has been previously investigated numerically in a simplified simulator using explicit FDM (Lindblom and Nordell (2006, 2007)) for both drinking water and irrigation applications. It was then found that cooling warm humid air in a non-perforated pipe resulted in a daily water production of $1.8 \text{ kg m}^{-1}\text{d}^{-1}$ (Lindblom and Nordell (2006)). Using perforated drainage pipes, under similar circumstances resulted in an irrigation yield of $2.3 \text{ mm}^{-1}\text{d}^{-1}$ (Lindblom and Nordell (2007)). To verify these results, and to make the model more flexible, the CI2D simulator was created, using implicit Galerkin finite elements (Lindblom et al. (2012)).

It is the objective of this study to use the validated CI2D model to demonstrate the potential of a theoretical CI system, and to identify the most crucial design parameters of such a system. Evaporation capacities of regular solar stills have been extensively studied for many designs (Tiwari et al. (2003a,b); Arjunan et al. (2009)), and is not included in this paper.

2 The CI2D model

The simulation model CI2D is a two-dimensional two-phase non-isothermal simulation program that was developed for simulating water, air, vapor, and heat transport in unsaturated soil (Lindblom et al. (2012)). The model has been validated with isothermal experimental data (Celia and Binning (1992); Touma and Vaucelin (1986)) and verified by the commercial software TOUGH2 (Emmert et al. (1995); Pruess et al. (1999); Pruess (2004)). In both comparisons, the CI2D model produced accurate results with excellent mass balance. The verification case (Emmert et al. (1995)) simulated hot air and vapor being injected into an initially dry, cool sand. This example is in principle very similar to condensation irrigation, and thus serves well to verify the underlying theory of current simulation model.

2.1 Assumptions

In the CI2D model (Lindblom et al. (2012)) the following assumptions were made for the calculations of the soil matrix: the soil gas relative humidity is approximated to $\Phi \approx 100\%$ (Pruess (1987)), because this value seldom falls below 99% in a porous medium under normal conditions. Air solubility in the water may be described by Henry's law (Bergman et al. (2011)), but since the effects of this fraction of dry air is quite small, the amount was neglected. Furthermore, effects of hysteresis, organic and chemical reactions, and mechanical dispersion were neglected in the model. Also, due to the low flux densities, local thermal equilibrium was assumed in all phases. The liquid phase was

assumed to be incompressible, and the solid matrix a rigid body with constant porosity.

The underground pipe used in the simulations were smooth, and during irrigation all condensate formed in the pipe were assumed to instantaneously and homogeneously transfer to the surrounding soil, although it is likely that the majority of the liquid water would infiltrate through the lower part of the pipe. Both air and vapor were assumed to behave as ideal gases, and the humidity was set to 100% in the pipe. Above ground, the ambient air temperature varied according to a sine function between 12 and 28°C, with constant wind speed and atmospheric pressure. The solar radiation varied corresponding to a latitude of 35°N, with a surface albedo of 0.3, and 50% of the surface assumed to be shaded by plant leaves. Lastly, the groundwater table was assumed to be deep enough so that capillary rise did not reach into a simulated section.

The mass and heat transfer in the soil along the pipe direction was assumed to be negligible. For increasing robustness and simulation efficiency, the model was therefore constructed as a series of two-dimensional cross-sectional meshes, connected via the pipe flow, running through all 2D meshes in the pipe nodes.

2.2 Model Description

The mass and heat transfer in the unsaturated soil was described by one balance equation for each of the three inherent components, water, air, and heat. The gas phase is frequently neglected in isothermal modeling of water movements in unsaturated soil (e.g. Celia et al. (1990); Forsyth et al. (1995); Gärdenäs et al. (2005); Ramos et al. (2012)), which is usually an acceptable simplification. However, in cases where gas may be captured in the soil, by waterfronts, or other barriers, the gas is unable to make room for any advancing water mass. The gas phase will then affect the water movement in the soil, and should be included in the calculations (Touma and Vaucelin (1986)).

When temperature gradients are considerable, the gas phase transport must never be neglected, since one of the most important mass and heat transfer mechanisms is the vapor transport of both latent and sensible heat from warm areas to cooler.

The air component in the gas phase is transported through the soil by diffusion within the gas and by gas phase advection, so the air mass is balanced as (e.g. Binning (1994); Helmig (1997)):

$$\frac{\partial}{\partial t} (S_g \theta \rho_a) + \nabla \cdot (\mathbf{i}_a + \rho_a \mathbf{v}_g) = f_a \quad (2.1)$$

in which the term f_a (kg s⁻¹ m⁻³) contains any sources or sinks within the control volume, $S_g \theta \rho_a$ (kg m⁻³) equals the mass of air per unit volume, $\mathbf{i}_a$ (kg

$\text{s}^{-1} \text{m}^{-2}$) is the air diffusion, ρ_a (kg m^{-3}) is the air density, and $\mathbf{v}_g$ (m s^{-1}) is the Darcy flux of the gas phase.

Water in the soil occurs in both liquid and gaseous phase. The liquid water is transported by advective motion, and vapor via advective gas motion and binary diffusion within the gas phase. Depending on the magnitude of the state parameters in a control volume, some water may shift phase through evaporation or condensation. The balance equation of water therefore includes water in both liquid and the vapor phase, so that phase transitions of water does not influence the total mass balance:

$$\frac{\partial}{\partial t} (S_w \theta \rho_w) + \frac{\partial}{\partial t} (S_g \theta \rho_v) + \nabla \bullet (\mathbf{i}_v + \rho_v \mathbf{v}_g) + \nabla \bullet (\rho_w \mathbf{v}_w) = f_w. \quad (2.2)$$

In the above, f_w ($\text{kg s}^{-1} \text{m}^{-3}$) represents any liquid or vapor source or sink, $S_w \theta \rho_w$ (kg m^{-3}) and $S_g \theta \rho_v$ (kg m^{-3}) are the mass of liquid and vapor per unit volume, $\mathbf{i}_v$ ($\text{kg s}^{-1} \text{m}^{-2}$) is the vapor diffusion, ρ_v (kg m^{-3}) is the vapor density, ρ_w (kg m^{-3}) is the water density, and $\mathbf{v}_w$ (m s^{-1}) is the flux density of the liquid phase.

The heat transfer through the porous media, $\mathbf{q}$ (W m^{-2}), occurs through heat conduction together with gas and liquid movements, but is greatly enhanced by the latent heat carried within the vapor. The energy balance for a control volume of a porous media is hence expressed as:

$$\frac{\partial Q}{\partial t} + \nabla \bullet \mathbf{q} = f_e$$

in which f_e (W m^{-3}) represents any heat source or sink, and Q (J m^{-3}) is the internal energy of the components comprising the control volume, according to:

$$Q = \theta \left[u_w S_w \rho_w + (1 - S_w) (u_v \rho_v + u_a \rho_a) + \frac{(1 - \theta)}{\theta} u_p \rho_p \right].$$

in which u_a , u_v , u_w , and u_p (J kg^{-1}), are the internal energy of air, vapor, water, and solid particles. ρ_p (kg m^{-3}) is the density of the solid.

The generalized Darcy flux, $\mathbf{v}_\alpha$ (m s^{-1}), describes the flux density of phase α by separating the total pressure gradient into an external pressure H_α (m) and potential (gravitational) pressure:

$$\mathbf{v}_\alpha = - \frac{k_{r\alpha} k_i \rho_w 0g}{\mu_\alpha} (\nabla H_\alpha + \nabla y) \quad \alpha = g, w, \quad (2.3)$$

in which y is the vertical direction, μ_α (Pa s) and $k_{r\alpha}$ (kg m^{-3}) are the viscosity and relative permeability of either the gas (g) or the liquid (w) phase, and k_i is the intrinsic permeability of the soil. The water pressure, H_w (m), is defined as

the sum of the gas phase pressure, H_g (m), and the negative capillary pressure, H_c (m), which in this model is estimated from the water retention function, as described by Van Genuchten (1980):

$$H_c = -\frac{1}{\rho_w 0 g a} (S_e^{1/m} - 1)^{1/n} \quad (2.4)$$

in which a , n , and $m = 1 - \frac{1}{n}$ are soil specific shape parameters and

$$S_e = \frac{S_w - S_r}{1 - S_r} \quad (2.5)$$

where S_r is the residual water saturation.

The relative permeabilities k_{rw} and k_{rg} are derived from the theory by Van Genuchten (1980) in conjunction with Mualem (1976):

$$k_{rw} = S_e^{1/2} \cdot \left(1 - (1 - S_e^{1/m})^m\right)^2, \quad (2.6)$$

$$k_{rg} = (1 - S_e)^{1/3} (1 - S_e^{1/m})^{2m}. \quad (2.7)$$

Concentration gradients arise in the gas due to gas pressure or temperature gradients, or as a result of condensation or evaporation. The gas diffusion of either dry air (a) or vapor (v), $\mathbf{i}_\beta$ ($\text{kg m}^{-2} \text{s}^{-1}$), is described by Fick's law according to:

$$\mathbf{i}_\beta = -\tau \rho_g S_g \theta D_0 \nabla X_\beta \quad \beta = a, v,$$

in which D_0 is the binary diffusion coefficient of air and vapor, τ is the tortuosity of the pores (Koorevaar et al. (1983)), and $\rho_g S_g \theta$ ($\text{m}^3 \text{m}^{-3}$) is the gas content (Lindblom et al. (2012)).

In the CI2D model, the heat flux, $\mathbf{q}$ (W m^{-2}), through a control volume of soil is a result of the water and gas advection and diffusion through the pore system together with heat conduction, according to:

$$\mathbf{q} = h_a (\rho_a \mathbf{v}_g + \mathbf{i}_a) + h_v (\rho_v \mathbf{v}_g + \mathbf{i}_v) + h_w \rho_w \mathbf{v}_w - k_h \nabla T. \quad (2.8)$$

In Eq. (2.8), h_a , h_v , and h_w (J kg^{-1}), are the enthalpies for air, vapor, and water, respectively, and k_h (W m^{-2}) is a local overall thermal conductivity for the porous media.

2.2.1 Mass and Heat transfer at the Surface

The example simulations in this study used solar radiation data, G_{sol} (W m^{-2}), corresponding to Malta (35°N) in May, using a surface albedo of 0.3. Above

ground, the ambient air temperature, T_a ($^{\circ}\text{C}$), was assumed to vary according to:

$$T_a = 8 \cdot \sin\left(\frac{2\pi t}{3600 \cdot 24}\right) + 20$$

in which t (s) is the time. The leaf canopy shaded 50% of the ground surface from incoming and outgoing radiation. The wind speed at the surface (1.7 m s^{-1}) corresponded to a constant, local convection coefficient of $12 \text{ W m}^{-2} \text{ K}^{-1}$ (Palyvos (2008)), and the long-wave radiation from the surface, G_{sky} (W m^{-2}), was:

$$G_{sky} = \sigma \cdot \left((\bar{T}_{surface} + 273.15)^4 - (T_{sky} + 273.15)^4 \right)$$

in which $\sigma = 5.67 \cdot 10^{-8} \text{ (W m}^{-2} \text{ K}^{-4})$ is the Stefan-Boltzmann constant, $\bar{T}_{surface}$ ($^{\circ}\text{C}$) is the mean surface temperature at the moment in question, and T_{sky} ($^{\circ}\text{C}$) is the effective sky temperature.

The evaporation of liquid water from the ground surface was in this model defined by the pressure potential difference between the surface nodes and the ambient, in which the latter, H_{∞} (m), can be estimated using the Kelvin-Laplace equation. However, because H_{∞} usually has a modest influence on the evaporation rate, and because the relative humidity is unknown in the simulations, the pressure potential was set to a constant value of -1000 m (Lappala et al. (1987)). The bare soil evaporation, q_s (kg s^{-1}), at surface node j became:

$$q_{s,j} = \frac{k_{rw,j} \rho_{w,j} k_i}{\mu_{w,j}} \cdot \Delta y \cdot (-1000 - H_{w,j}) \quad (2.9)$$

in which Δy is the vertical distance between the two topmost nodes in the section.

Part of the vapor flux from the pipe leaves through the ground surface, thereby contributing to the surface water losses. Since the ambient air pressure was assumed constant at atmospheric pressure, the gas flux of air and vapor through the surface depended on the soil humidity, temperature and gas pressure.

If the ground is vegetated, surface evaporation will be enhanced by plant transpiration. The combined surface and vegetation evaporation is sometimes called evapotranspiration (ET). The water supply to the transpiration is taken from the liquid water in the soil near the plant roots. Very little of the water taken up by the roots is actually stored inside the plant, most is lost through transpiration (Robinson and Ward (2000)). The root water uptake is hence a water loss from the ground to the ambient as well.

In the vicinity of the plant roots, water is transported towards the roots

by the negative water pressure at the root surfaces. The magnitude of this negative pressure varies according to crop and soil depth, but is usually set to the permanent wilting point of the specific plant, defined as the soil pressure potential at which plants wilt. Lappala et al. (1987) used the following method for determining plant transpiration:

The water extraction by the roots was divided among the “root zone nodes” in the model according to the root activity function, $r(y, t)$ (m m^{-3}). The root extraction, q_m (kg s^{-1}), is determined for any node j in the root zone by:

$$q_{m,j} = \rho_{w,j} v_j \frac{k_i k_{rw,j} \rho_{w0} g}{\mu_{w,j}} r(j, t) (H_{root} - H_{w,j}), \quad (2.10)$$

in which v_j is the volume of the specific element, and $H_{root} = -150$ m by default in the current model simulations Lappala et al. (1987). The total ET at node j is then:

$$ET_j = q_{s,j} + q_{m,j}$$

The potential evapotranspiration, PET (mm month^{-1}) is the estimated maximum evapotranspiration rate, that would occur if sufficient water were available in the topsoil. It can easily be estimated by the Thornthwaite model (Kumar et al. (1987)), since it only takes the average temperature and day length into account (Robinson and Ward (2000); Kumar et al. (1987)):

$$PET = 16 \left(\frac{dl}{12} \right) \left(\frac{10T_a}{I} \right)^a \quad (2.11)$$

in which dl (hours) is the average day length of the period considered, T_a ($^{\circ}\text{C}$) is the mean ambient temperature, I is a heat index, calculated as

$$I = \sum_{i=1}^{12} \left(\frac{T_a}{5} \right)^{1.514}$$

and the exponent a in Eq. (2.11) is

$$a = 6.75 \cdot 10^{-7} I^3 - 7.71 \cdot 10^{-5} I^2 + 1.792 \cdot 10^{-2} I + 0.49239$$

As long as the actual evaporation rate was lower than PET , this value was used. If ET exceeded PET , then PET was used instead. In Condensation Irrigation, the soil surface is kept relatively dry, even during irrigation, which leads to lower surface water content than in conventional irrigation systems. This means that PET very seldom was reached.

2.2.2 Pipe flow

While flowing through the pipes, the air is cooled by the ground and vapor condenses to form liquid freshwater. In perforated pipes, this water and some humid air infiltrate the surrounding soil, thereby decreasing the mass flux through the pipes. Applying a control volume around the pipe surface and in the pipe direction through locations $i-\frac{1}{2}$ and $i+\frac{1}{2}$ before and after soil section i , the airflow heat loss in a horizontal pipe, $q_{p,i}$ (W) was:

$$q_{p,i} = \dot{m}_l \Big|_{i-\frac{1}{2}} \left(h_l + \frac{c^2}{2} \right) - \dot{m}_l \Big|_{i+\frac{1}{2}} \left(h_l + \frac{c^2}{2} \right)$$

where $\dot{m}_l$ (kg s^{-1}), h_l ($\text{J kg}^{-1} \text{ }^\circ\text{C}^{-1}$), and c (m s^{-1}) are the mass flux, enthalpy, and velocity of the humid airflow. The heat transferred from the pipe wall to the soil in the two-dimensional section i , $q_{p,i}$ (W), also equals the energy change of the airflow, i.e. the sum of convection heat transfer due to temperature differences and the rate at which heat is lost with the mass transfer through the pipe perforations to the soil in section i :

$$q_{p,i} = A_p (h_D (T_l - T_p) + L h_m (\rho_l - \rho_p))_i + \dot{m}_{s,i} h_{l,i}$$

where A_p (m^2), T_p ($^\circ\text{C}$), and ρ_p (kg m^{-3}) are the pipe surface area, wall temperature and vapor density, T_l ($^\circ\text{C}$) and ρ_l (kg m^{-3}) is the mean air temperature and density in the cross-section, and $\dot{m}_{s,i}$ (kg s^{-1}) is the mass flux through the perforations. The convection coefficient, h_D ($\text{W m}^{-2} \text{ }^\circ\text{C}^{-1}$), is:

$$h_D = \frac{Nu_D \cdot k_l}{D_p}$$

in which k_l ($\text{W m }^\circ\text{C}^{-1}$) and D_p are the thermal conductivity of air and the pipe diameter. The Nusselt number, Nu_D , for turbulent pipe flow was estimated from the correlation (Bergman et al. (2011))

$$Nu_D = \frac{\frac{f}{8} \cdot (Re_D - 1000) \cdot 0.71}{(1 + 12.7 \cdot (f/8)^{0.5}) \cdot (0.71^{2/3} - 1)},$$

in which Re_D is the Reynolds number, and f is the friction factor, given as

$$f = (0.79 \cdot \ln(Re_D) - 1.64)^{-2}.$$

The mass convection coefficient, h_m (m s^{-1}), was derived from the Reynolds analogy (Bergman et al. (2011)):

$$h_m = \frac{h_D D_{av}}{\rho_l^{1/3} c_p^{1/3} k_l^{-2/3}}.$$

In moving from location $i-\frac{1}{2}$ to $i+\frac{1}{2}$, the mass flow is reduced by the amount of vapor condensing in the control volume and by the mass of humid air leaving through the perforations, $\dot{m}_{s,i}$ (kg s^{-1}), in section i :

$$\dot{m}_{l,i+\frac{1}{2}} = \dot{m}_{l,i-\frac{1}{2}} - A_p h_{m,i} (\rho_{l,i} - \rho_{p,i}) - \dot{m}_{s,i},$$

in which A_p (m^2) is the pipe surface area. $\dot{m}_{s,i}$ was assumed proportional to the difference in the static pressure of the humid air in the pipe, $H_{l,i}$ (m), and the gas pressure in the soil outside the pipe, $H_{g,i}$ (m), at that section, according to:

$$\dot{m}_{s,i} = -A_h k_i \rho_{l,i} g \rho_{w0} \sum_{j=1}^n \frac{k_{rg,j}}{\mu_{g,j}} (\nabla H_{g,jk} + \nabla y)$$

where A_h (m^2) is the total perforated area of the pipe in the section. $\nabla H_{\alpha,jk}$ is the pressure gradient between the air flow static pressure in section i , node k , and the adjacent pipe wall nodes j in the same section i .

2.3 Initial and Boundary Conditions

Each two-dimensional section of the soil was divided into subsections, according to Fig 2.1. To improve stability of the simulations, the node spacing was chosen so that every control area was square shaped with dimension $\Delta x = \Delta y = 0.1$ m. The distance between the sections were $\Delta z = 1.0$ m. In Fig. 2.1 the node numbering and location of the specific boundary conditions are defined. The values used for the root activity function are given at each node included in the root zone.

As seen in Fig. 2.1, one vertical mirror line was placed through the location of the crop roots, and another through location of the mid-section of the pipe. This means that only half of the mass and heat transfer from one pipe entered the simulated section.

In each of the calculated 51 sections along a pipe, the nodes at the ground surface, pipe wall, and plant roots were contained within the vectors **top**, **pipe**, and **roots**, respectively:

$$\begin{aligned} \mathbf{top} &= [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7]^T \\ \mathbf{pipe} &= [22 \ 23 \ 30 \ 36 \ 37]^T \\ \mathbf{roots} &= [14 \ 21 \ 28 \ 35 \ 42 \ 49 \ 56 \ 63]^T \end{aligned}$$

At the soil surface, water, vapor, air, and heat are exchanged with the atmosphere. Since the atmospheric gas pressure was known, the mass balance equations, Eqs. (2.1-2.2), could be used to find the air and vapor fluxes from

the soil to the ambient. The liquid water lost to the atmosphere by evapotranspiration is the sum of Eqs. (2.9) and (2.10).

During the daily irrigation, humid air, freshwater, and heat were injected from the pipe boundary into the soil. At night, the soil is cooled by the cool air flux inside the pipe and the gas flux through the pipe perforations.

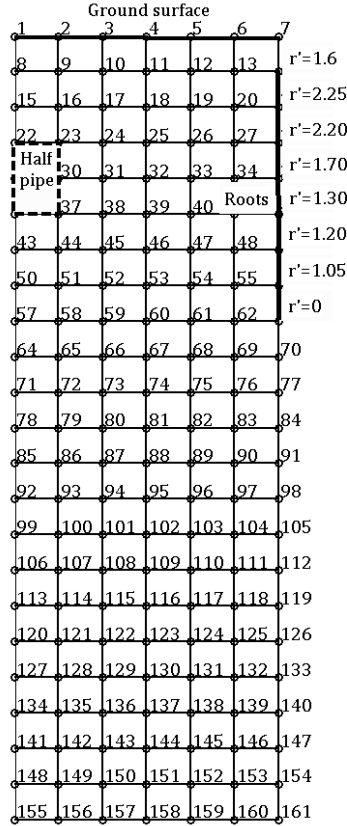


Figure 2.1: Vertical section of the simulated two-dimensional soil-pipe system, including node numbering. The nodes containing Neumann boundary conditions are marked in the grid. The root activity function is given as $r = r' \cdot 10^4 \text{ m m}^{-3}$.

In the simulations, the initial soil temperature was set to 20°C, and the soil gas pressure was at atmospheric level. The soil saturation was set to 0.2, just above the residual saturation of 0.19, and the water pressure was calculated from the capillary and gas phase pressures (Eq. 2.4). Other parameters for the soil in the reference case was adopted from measurements done by Brooks and Corey (1964) on a fine sand. These parameters are given in Table 1, together with the inlet airflow parameters and the pipe configuration:

Table 1: Parameters used in the simulated standard case. a and m are curve fitting parameters used for the van Genuchten correlation (Eq. 2.4) of the soil water retention curve, θ is the porosity, and k_i is the intrinsic permeability. T_l , p_l , and $\dot{m}_l$ are the inlet airflow temperature, pressure, and mass flux. The diameter, burial depth, and spacing of the pipes are denoted by D_p , d , and cc .

Soil parameters			Inlet airflow parameters			Pipe configuration		
a	0.96	m	T_l	70	°C	D_p	0.2	m
m	6.9		p_l	106330	Pa	d	0.4	m
θ	0.377	m ³ m ⁻³	$\dot{m}_l$	0.5	kg s ⁻¹	cc	1.2	m
k_i	$1.745 \cdot 10^{-12}$	m ²						

3 Results and Discussion

The reference CI system was simulated over a period of 30 days. During this time, the mean condensation rate and vapor flux in one pipe developed according to Fig. 3.1. The latter decreases during the first week because of the increasing soil water content around the pipe wall. As the soil pores became increasingly water filled, gas transport was hindered, which caused a gas pressure build up around the pipe. When the gas pressure rose high enough to push through the wetted soil pores the gas flux to the surface reached its diurnal steady state.

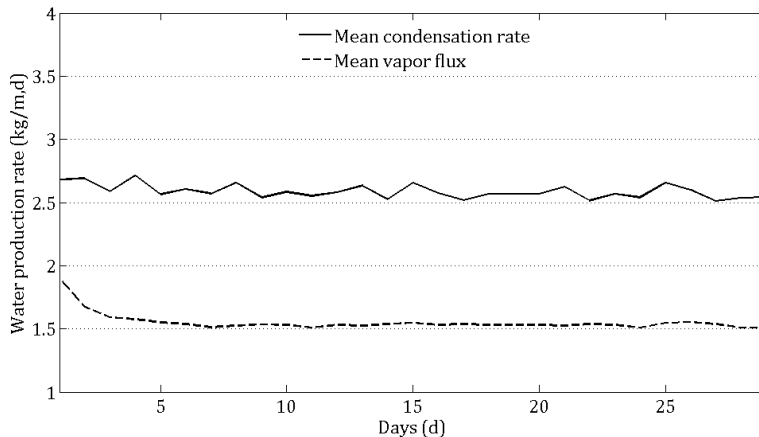


Figure 3.1: Condensation rate inside perforated pipe and vapor flux through pipe perforations during 30 days. Mean values along 50 m pipe.

During the first 30 days, the average daily irrigation yield due to pipe condensation was $2.16 \text{ mm}^{-1} \text{ d}^{-1}$ ($2.59 \text{ kg m}^{-1} \text{ d}^{-1}$), and the vapor flux contribution

was $1.29 \text{ mm}^{-1} \text{ d}^{-1}$ ($1.55 \text{ kg m}^{-1} \text{ d}^{-1}$), resulting in a total average irrigation yield of $3.44 \text{ mm}^{-1} \text{ d}^{-1}$.

Because the airflow was cooled by the ground, the condensation rate declined along the pipe. At diurnal steady state, the condensation rate decreased from $3.80 \text{ kg m}^{-1} \text{ d}^{-1}$ to $1.70 \text{ kg m}^{-1} \text{ d}^{-1}$ along the pipe, and the vapor flux decreased from $2.32 \text{ kg m}^{-1} \text{ d}^{-1}$ to $0.95 \text{ kg m}^{-1} \text{ d}^{-1}$ (Fig. 3.2). Irrigation was hence more efficient near the pipe entrance (5.1 mm d^{-1}) than after 50 m (2.2 mm d^{-1}).

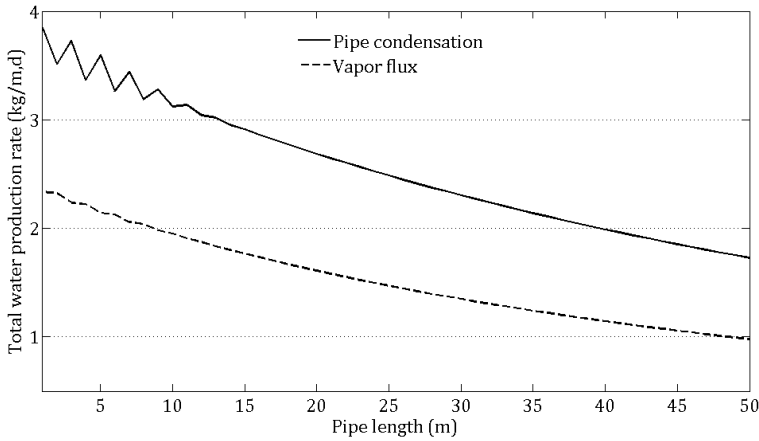


Figure 3.2: Condensation rate along the first 50 m of a drainage pipe after 30 days of irrigation

During the first 30 days of irrigation, 3100 kg of water was released from the half pipe to the soil in the simulated section (Fig. 2.1), of which 1940 kg was liquid water, and 1160 kg was vapor. Of this, 125 kg water was lost through deep percolation, 613 kg was taken up by plant roots, 1466 kg was accumulated in the ground control volume, and 896 kg was lost through the surface. The resulting mass balance of water over all 51 sections over 30 days becomes:

$$\varepsilon_w = \frac{3100 - 125 - 613 - 1466 - 896}{3100} = -2.4 \cdot 10^{-4}.$$

Of the surface loss, 344 kg was vapor, and 552 kg was liquid water evaporation. This means that 70% of the vapor entering the soil with the humid air flux formed freshwater in the soil and contributed to irrigation.

The amount of dry air injected through the pipe perforations was 15995 kg, but 15998 kg exited through the soil surface. 3 kg of the original air content was evacuated from the ground due to the increasing water content of the soil. The mass balance for air is then:

$$\varepsilon_a = \frac{15995 - 15998 + 3}{15995} = -1.7 \cdot 10^{-5}.$$

After 30 days of operation, the humid airflow had transferred $5.25 \cdot 10^9$ J of heat to the soil, of which $2.47 \cdot 10^9$ J was released to the atmosphere, $1.12 \cdot 10^9$ J was transported below the control volume, and $7.42 \cdot 10^7$ J was used by the plant roots. The ground heating amounted to $1.57 \cdot 10^9$ J. By these figures, the energy balance is:

$$\varepsilon_e = \frac{5.25 \cdot 10^9 - 2.47 \cdot 10^9 - 1.12 \cdot 10^9 - 7.42 \cdot 10^7 - 1.57 \cdot 10^9}{5.25 \cdot 10^9} = 2.8 \cdot 10^{-3}.$$

Of the vapor flux from the pipe, only 30% was transported with the air to the surface, the rest condensed in the ground. During the 12 hours when $T_a > 20^\circ\text{C}$, the ground was irrigated and heated from the buried perforated pipe. Solar irradiation resulted in a net radiation heat transfer to the ground surface during this time as well. Fig. 3.3 shows the water and vapor fluxes from the soil surface together with the ambient air and surface temperatures, and the temperature difference between the surface and air.

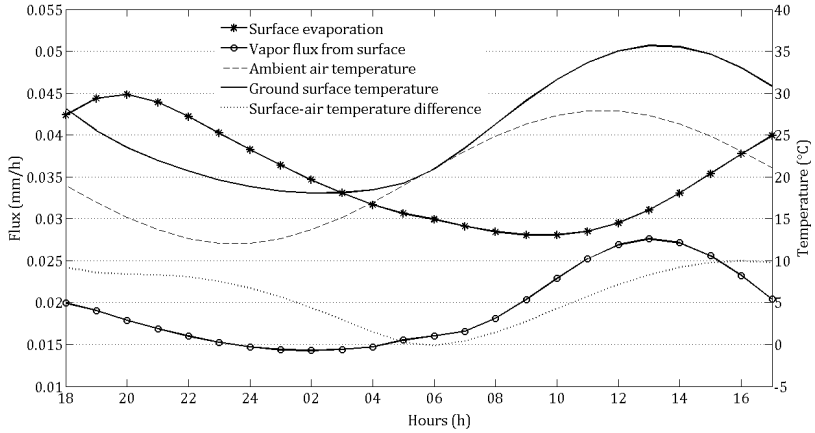


Figure 3.3: Surface water loss through evaporation and vapor transport to the ambient together with ambient air and ground surface temperature.

Because gas flux from the pipe is the main heat transfer mechanism in the soil, the surface temperature and vapour flux from the surface are strongly correlated with each other: during the daily irrigation, the gas flux from the pipe heats the ground surface. The vapour content of the gas flux is relatively high, causing a high vapour loss through the ground surface. At night, cool ambient air transfers from the pipe through the ground. This cool air contains

less vapour, causing less vapour loss through the ground surface.

The hourly liquid surface evaporation varies between 0.028 mm h^{-1} and 0.045 mm h^{-1} , with the maximum occurring at 9 o'clock at night. The reason for this unusual evaporation cycle is that cool night air coming through the pipe perforations is heated by the wet soil around the pipe. As the air becomes warmer, it picks up liquid water from around the pipe so that the air becomes vapour saturated (the relative humidity is assumed to be 100%). The formed vapour is carried towards the surface by the air flux, which is now being cooled by the cooler topsoil. As a consequence, some of the vapour carried by the air is condensed once more near the ground surface, from where it finally evaporates into the ambient air.

Under normal circumstances, the ground is cooled at night by sky radiation, so that air near the surface cools and vapour precipitates as water on the ground during night. In the CI system, the ground stores the daily heat release from the pipe. The ground surface therefore remains warmer than the ambient air, and surface evaporation occurs.

Another factor that increases nightly evaporation in the CI system is that the cool night air flux from the pipe increases the gas pressure in the soil due to the denser air. As the air above the pipe is more efficiently transported to the surface, liquid water above and around the pipe is lifted towards the surface as well, which further increases evaporation.

The nightly upward water flux becomes apparent when comparing saturation profiles at 6 and 9 o'clock at night (Figs. 3.4-3.5). Water saturation around the pipe is reduced as the cool air flux transports water to the surface. As a result the temperature above the pipe is reduced, but the temperature in the root zone, at the right vertical boundary, is increased during the first hours of the night, due to mass and heat transfer from the pipe region. Figs. 3.4-3.5 also show how the gas phase pressure is increased at night around and below the pipe, retarding downward motion of water somewhat.

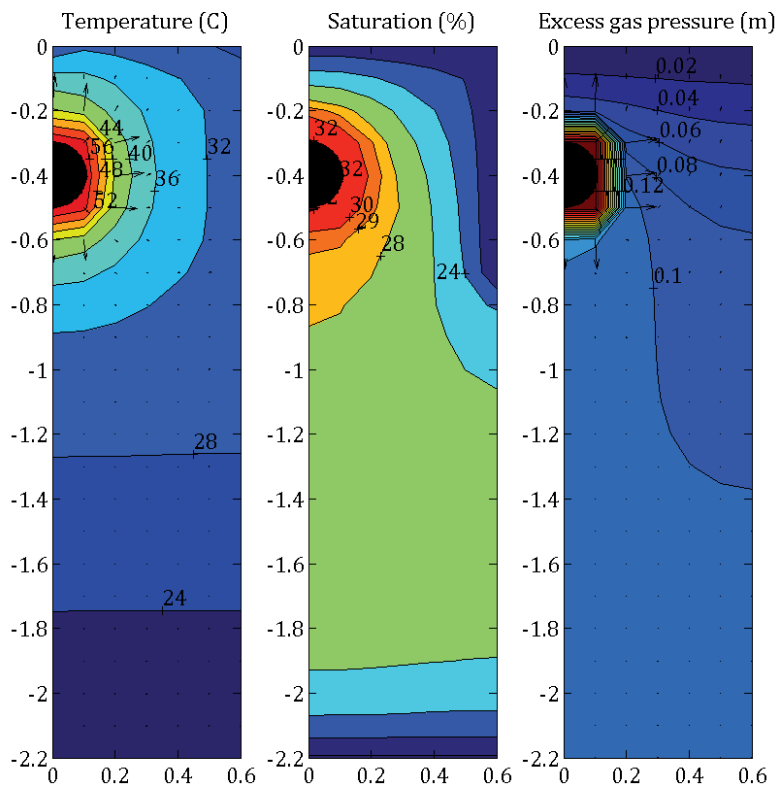


Figure 3.4: Temperature profile including liquid water flux, water saturation degree, and gas phase flux with gas phase flux, 25 m along the pipe, at 18.00 in the afternoon, just before irrigation stops for the day.

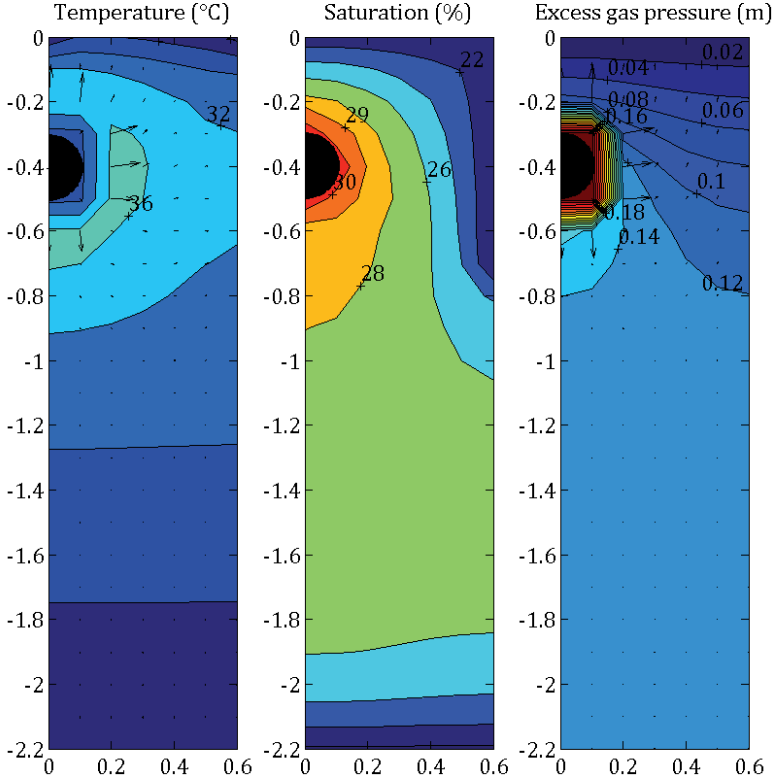
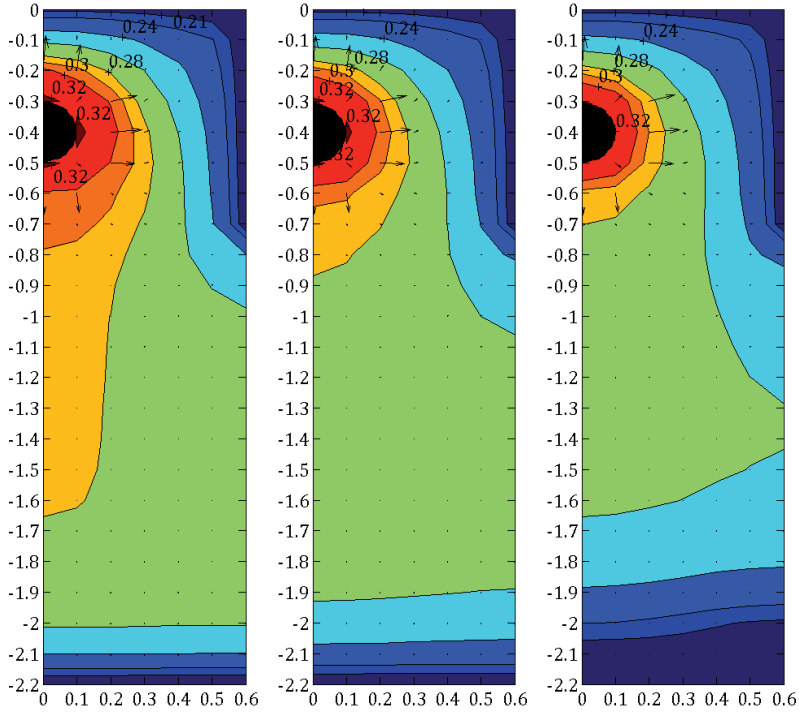


Figure 3.5: Temperature profile including liquid water flux, water saturation degree, and gas phase flux with gas phase flux, 25 m along the pipe, at 21.00 at night.

Over the first 30 days, the mean surface evaporation rate was 0.61 mm d^{-1} , and the mean vapor flux 0.38 mm d^{-1} . By adding the mean daily root transpiration of 0.68 mm d^{-1} , the average evapotranspiration rate was 1.67 mm d^{-1} . When using subsurface irrigation, evaporation losses are always reduced due to the comparatively dry soil surface. Zotarelli et al. (2009) measured a daily evapotranspiration rate of about $2.5 - 3.0 \text{ mm d}^{-1}$ when subsurface irrigation was used. In their experiments on tomato growth in a sandy soil, the water consumption using subsurface drip irrigation ranged from 2.6 to 4.6 mm d^{-1} .

Fig. 3.6 displays the water saturation and the direction of the liquid water flux at the end of the daily irrigation on the 30th day in sections 10 m, 25 m, and 40 m from the pipe inlet.

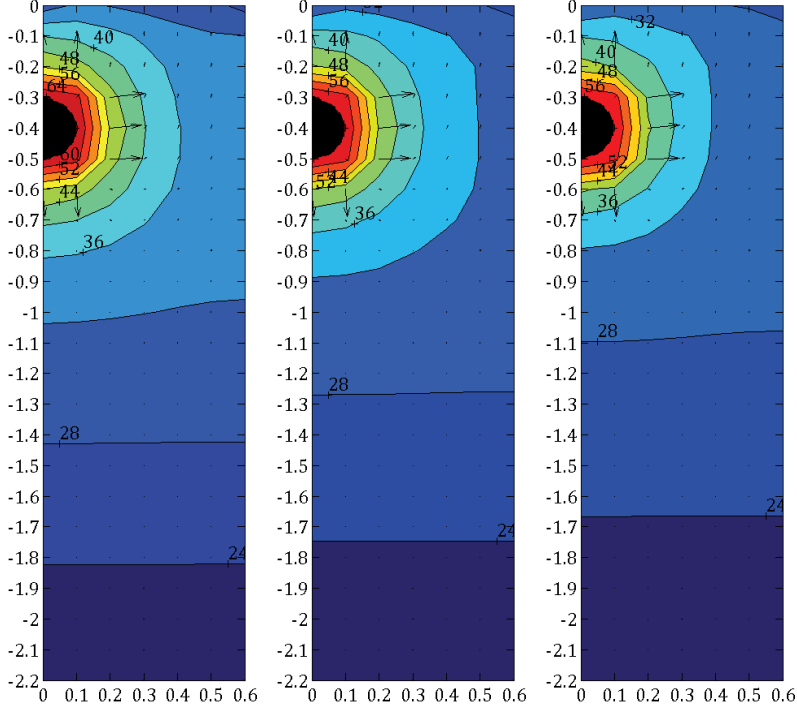


(a) 10 m from pipe inlet (b) 25 m from pipe inlet (c) 40 m from pipe inlet

Figure 3.6: Saturation profiles and liquid water flux along the pipe after 30 days of irrigation. The plant roots extend to 0.8 m below the surface at the right vertical boundary.

Due to the sandy soil used as reference, there is a noticeable downward concentration of liquid water. The roots along the vertical right border efficiently remove water transported to the root zone, leaving this area dry. In common subsurface drip irrigation systems, the main water loss is through deep percolation. In CI, this loss was somewhat reduced due to the upward directed water pressure gradient, which guided the water in a less steep path downward.

Temperature profiles after 30 days of irrigation are shown in Fig. 3.7 for the warmest time of day. The soil temperature in the root zone is equal to or below 32°C , regarded as acceptable for most crop roots. During the day, the pipe wall reaches temperatures well over 50°C , thus preventing any roots from growing into the pipe.



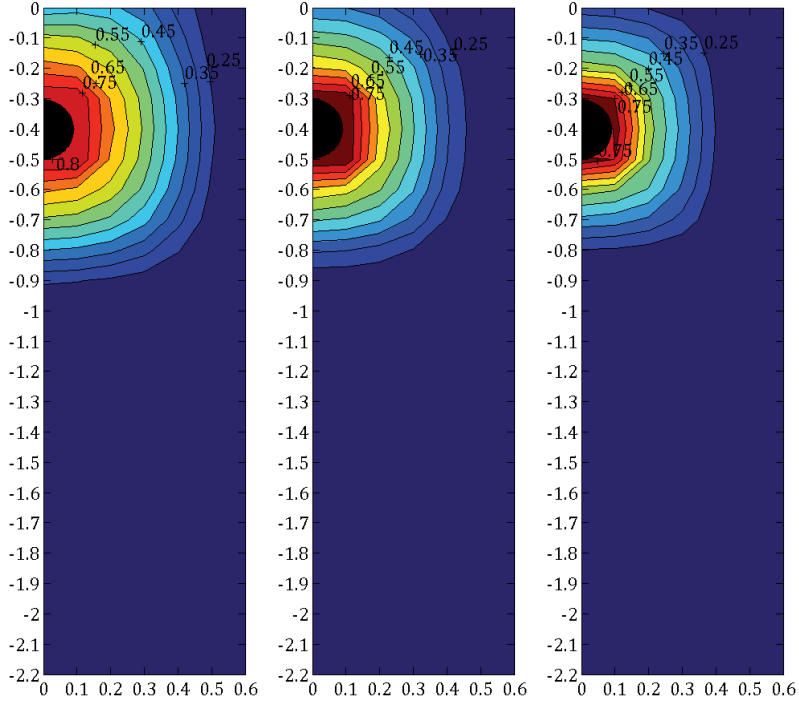
(a) 10 m from pipe inlet (b) 25 m from pipe inlet (c) 40 m from pipe inlet

Figure 3.7: Temperature profiles in the soil after 12 hours of irrigation on the 30th day of irrigation.

To compare the irrigation yield in different soils, the fine sand (Brooks and Corey (1964)) in the reference case was exchanged with yolo light clay (Haverkamp et al. (1977)). Fig. 3.8 illustrates the water saturation level after 30 days of irrigation. When comparing the section with the reference case (Fig. 3.6) it is evident that the water saturation around the pipe is now considerably increased. Gravity has almost no impact on the water distribution, and surface evaporation, root water uptake, and deep percolation are very small.

Of the 2362 kg water released, 99% accumulated in the soil in close vicinity of the pipe. This is characteristic for clays with high capillarity. In yolo light clay, the negative capillary pressure near residual saturation ($S_r+0.01$) is -3782 m, while in the reference sand, it is only -2.2 m.

The total irrigation amount in the clay soil was $2.6 \text{ mm}^{-1} \text{ d}^{-1}$. This considerable reduction in irrigation rate was a result of the meager vapor flux that



(a) 10 m from pipe inlet (b) 25 m from pipe inlet (c) 40 m from pipe inlet

Figure 3.8: Water saturation in yolo light clay in three sections of the soil along the pipe

managed to infiltrate the clay through the pipe perforations. The condensation rate inside the pipe was, however, increased relative to the reference case (see Table 2).

There are alternative uses for the principle of the CI system, such as drinking water production and soil humidification to prevent desertification. To estimate the capacity of these systems, the reference scenario was altered to simulate drinking water production by removing the perforations in the pipe wall, and to simulate soil humidification by removing crop roots. The latter may also serve to simulate initial stages of crop growth.

The resulting condensation rate inside the pipe and total water flux to the ground is presented in Table 2 for the reference, together with the described alterations, and yolo light clay.

Table 2: Comparison of the water production rate in different applications and soils. Condensation rate values equals the amount of vapor that condenses inside the pipe. Irrigation rate denotes the total amount of water added to the soil.

Application / soil type	Condensation rate (kg m ⁻¹ d ⁻¹)	Irrigation rate (mm d ⁻¹)
Reference CI	2.590	3.444
No roots in soil	2.590	3.439
No perforations	2.711	-
Yolo light clay	3.158	2.624

In Table 2, more vapor condenses inside a pipe in the drinking water application and when using a clay soil. This is because the humid air flux from the pipe increases the temperature in the ground, thus reducing its cooling capacity. The condensation rate in a pipe buried in a clay is higher than in a pipe without perforations in a dry sand, because the wet clay has a higher thermal conductivity than the dry sand. The total water production is, however, increased when using perforated pipes in a sand, due to the additional vapor flux condensing in the soil. When there are no plant roots in the soil, the irrigation rate is slightly reduced, since roots enhance the water flux, and thus the heat flux, away from the pipe.

Fig. 3.9 illustrates the irrigation rate as a function of inlet airflow temperature and pressure, and pipe depth and spacing. It appears that the irrigation rate is increased with increasing pressure, and reduced by increasing pipe depth.

However, although shallow pipe depths increase total water production, the shorter distance between the pipe and the ground surface also increases the vapor and water fluxes to the atmosphere, which reduces the water accumulated in the soil. Similarly, an increased inlet air pressure leads to higher irrigation yield, due to greater vapor flux, but the ground surface water and vapor flux is increased even more, which leads to somewhat less water remaining in the ground.

Even though the water production increases with spacing, the irrigation rate per unit area, (mm d⁻¹), is actually decreased, since the surface area also becomes larger.

The most efficient way to increase the water production inside the buried pipes is to increase the inlet airflow temperature (Lindblom and Nordell (2006, 2007)). The problem with this measure is that the temperature in the soil also increases, which may be harmful to the crop roots. To reduce the ground heating, the pipes can be placed wider apart, and/or buried at a shallower depth. This reduces the ground temperature everywhere in the soil section, which makes it possible for roots to develop closer to the pipes, in the more

humid area of the soil.

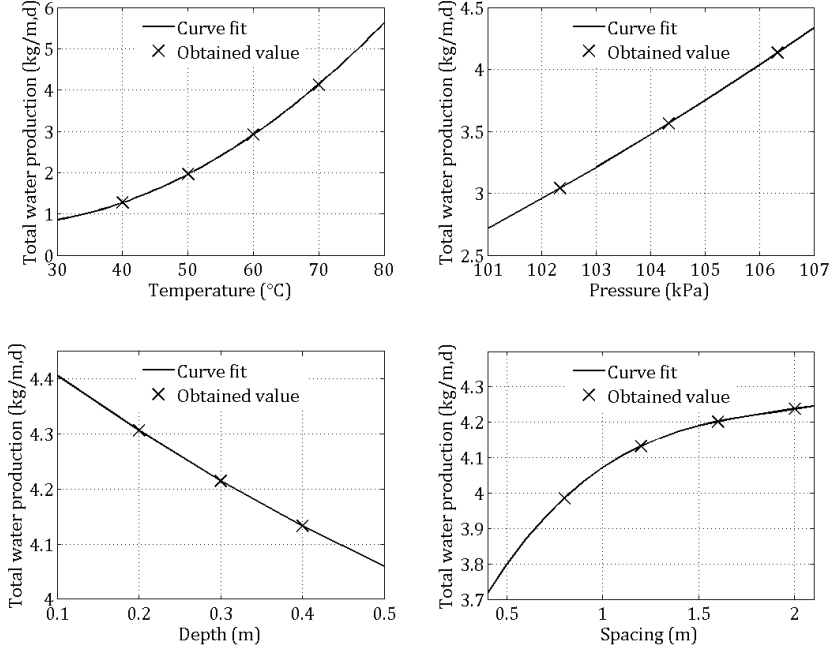


Figure 3.9: The irrigation yield as a function of inlet airflow temperature, inlet air pressure, piped depth and spacing.

When changing the temperature at the inlet of a pipe from 70°C to 60°C, the irrigation rate was reduced by 30%, and the total root water uptake nearly halved, from 613 kg to 354 kg. The soil temperature in the root zone was, on the other hand, reduced by 5°C over the first half of the pipe.

By reducing the spacing between the pipes to 0.8 m, the soil temperature in the root zone rose 5-8°C along the pipe, and the irrigation rate was increased to 5.0 mm d⁻¹. The root water uptake was also more than doubled (1377 kg) compared to the reference, which means that the soil saturation near the roots was significantly increased.

To obtain a higher root water uptake, the roots should hence be placed closer to the pipe. Since the soil temperature in this area is quite high in the reference case, the pipe spacing could be increased to compensate for this. Two rows of crop would then extract water from one pipe (Fig. 3.10), which means that the irrigation supply must be at least twice as big, and the soil temperature equal to or lower than the reference, in order to make this option more viable.

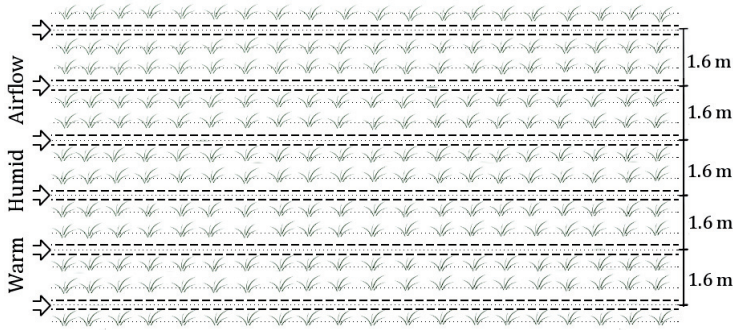


Figure 3.10: Alternative layout of the underground drainage pipes. Two rows of crop is planted between each drainage pipe.

The double crop row option was simulated with a pipe spacing and depth of 1.6 m and 0.4 m, respectively, and the inlet air temperature was maintained at 70°C. The plant roots were placed on either side of the pipe at a distance of 0.5 m from the pipe center. The resulting saturation and temperature of the mid-section of the pipe is shown in Fig. 3.11.

The resulting irrigation rate was now 2.64 mm d⁻¹ (4.22 kg m⁻¹ d⁻¹), but the root water uptake increased to 1469 kg, which is 2.5 times of that for the reference scenario. The temperature in the rooting zone was also kept below 32°C. In the reference case, 19.8% of the water from the pipes was used by the plants, while in the double crop row layout, this number increased to 46.6%. Evidently, the double crop row layout is more water efficient in terms of irrigation. Another advantage with this layout is that less pipes are required: in the reference case, 82 pipes per 100 m are required, while using double crop row, 62 pipes will cover that same area.

When high airflow temperatures are not achievable, plants must be able to grow close enough to the pipe in order to obtain enough irrigation. An example simulation of this was carried out using an inlet air temperature of 50°C, and placing the pipes 0.6 m apart. After 30 days, the temperature in the root zone was still acceptable, while the irrigation was reduced to 3.00 mm d⁻¹. The root water uptake was, however, increased compared to the reference level (645 kg), and 48% of the water supplied was used for water transpiration. This layout was hence more than twice as water efficient as the reference scenario.

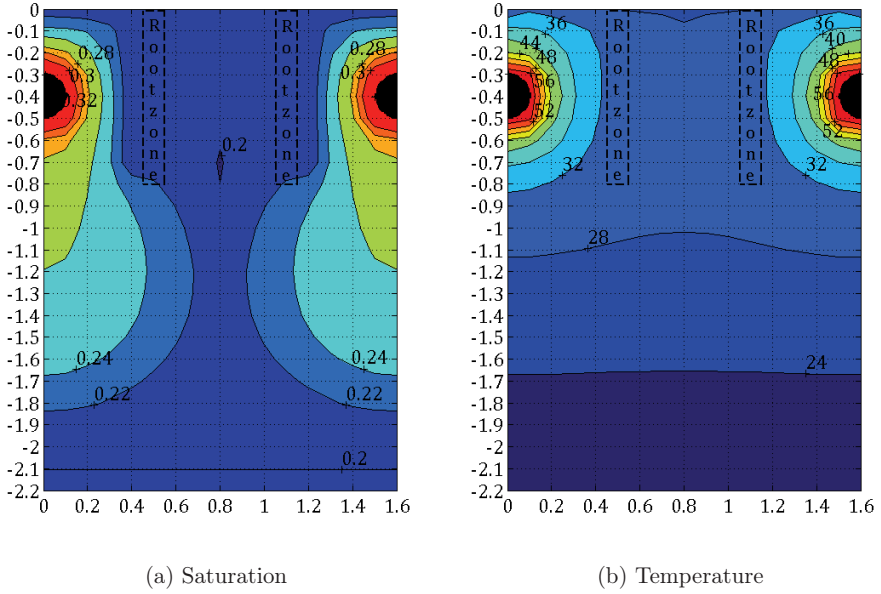


Figure 3.11: Water saturation and temperature profiles at 25 m along the pipe when using double crop rows.

4 Conclusion

In the condensation irrigation (CI) system, solar stills are connected to a sub-surface irrigation system, consisting of horizontal drainage pipes. The solar stills provide the drainage pipes with warm, humidified air, that condenses inside the pipes to form irrigation water. Some of the humid air leaks through the pipe perforations and contributes to irrigation and soil aeration.

A theoretical system was studied using the validated model CI2D (Lindblom et al. (2012)). The reference scenario simulated a cultivated field with a sandy soil in Malta during May. For the given set of inlet airflow properties, the irrigation rate was 3.44 mm d^{-1} . In average, crop roots consumed 0.68 mm d^{-1} , which is 19.8% of the applied water.

It was found that by planting the crops closer to the pipe, much higher root water uptake rates could be achieved. However, by decreasing the pipe spacing in the reference scenario, the temperature rose too high in the root zone. To overcome this, two alternative solutions were examined. The first solution used double rows of crop planted in between two pipes, using a pipe spacing of 1.6 m. As a result, the temperature dropped to below 32°C in the root zone, and crop water uptake rose 2.5 times that in the reference scenario,

corresponding to 25% higher root water uptake per crop row. The irrigation rate decreased compared to the reference scenario, due to the larger surface area associated with one pipe.

In the other solution, the inlet airflow temperature was reduced from 70°C to 50°C, and the pipes were placed at 0.6 m distance from each other. Due to the exponential temperature dependance of vapor content in air, the irrigation rate declined to 3.00 mm d⁻¹. Nevertheless, the soil temperature was within tolerance in the root zone, and the roots extracted as much as 48% of the added water, which corresponds to a higher root water uptake than in the reference case.

When little or no humid air infiltrates the soil through the pipe perforations, the condensation rate inside the pipes are increased, due to the lesser heat release to the ground. By simulating the reference scenario in a clay soil, the water condensing inside a pipe quickly saturated the soil around the pipe, thus blocking the humid air flux through the perforations. As a result, the condensation rate in the pipe increased, but the total water production decreased, due to negligible vapor contribution. The root water uptake was also less than 1% of the water released.

Because the airflow is cooled along the pipe, the irrigation rate decreases with distance from the inlet. The irrigation pipes should therefore be made short to maintain a high irrigation yield. To reduce the magnitude of the difference in irrigation supply, parallel pipes in the field should have opposite flow directions, implying air humidifiers on both sides of the irrigated field.

In experiments performed by Hausherr and Ruess (1993), condensation irrigation halved the crop water need of tomatoes. Assuming their results to be generally valid, crops normally requiring 6.9 mm d⁻¹ or less could be cultivated in the reference system.

To further augment the condensation rate inside the pipes, relatively cool feed water to the solar stills could be transported in cooling pipes, running inside the drainage pipes. These would then act as a cross-flow heat exchanger, pre-heating the water in the solar stills, and simultaneously improving the condensation rate of the airflow.

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Paper V

Experimental study of underground irrigation by condensation of humid air in perforated pipes

Lindblom, J., Nordell, B.

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Experimental Study of Underground Irrigation by Condensation of Humid Air in Perforated Pipes

Jenny Lindblom*, Bo Nordell

Department of Civil, Environmental and Natural resources engineering, Luleå
University of Technology, SE-97187 Luleå, Sweden. *E-mail: jeli@ltu.se,
Telephone: +46 920 493630

Abstract

A small scale Condensation Irrigation (CI) system was constructed to investigate the flow patterns of water, air and heat in the soil surrounding a perforated pipe from which water, heat and humid air was transferred. A 0.2 m long cross-section of sand and pipe was used to emulate a two-dimensional section of a CI system. Under these down-scaled conditions, the mean irrigation rate in the sand box was 1.03 mm d^{-1} . The major heat transfer mechanism in the sand profile was gas advection, which greatly reduced the sand temperature around the pipe. Nearly 50% of the vapour leaving the airflow inside the pipe, was transported to the sand surface by gas advection.

1 Introduction

Condensation Irrigation (CI) is a combined desalination and subsurface irrigation system, making use of saline or otherwise contaminated water for supplying clean irrigation. The system is divided into two parts: in the first, solar thermal energy evaporates non-potable water inside a solar still, and the formed vapour heats and humidifies ambient air flowing above the water surface inside the still. The warm and humid air is thereafter led into the second part of the system, comprising a horizontal array of underground drainage pipes.

While flowing through the pipes, the ground acts as a condenser, and cools the humid air so that vapour precipitates as freshwater inside the pipes. The perforations in the pipe wall enable the water to percolate into the the surrounding

soil, and thereby irrigate it. Some of the humid air also manage to leak through the perforations and contribute to increased irrigation due to vapour condensation in the sand.

To assess the irrigation yield and transport of water and heat in a soil surrounding a drainage pipe in the CI system, a down-scaled model of a buried drainage pipe was constructed inside a thermally insulated sand box, through which humidified warm air was pumped.

The airflow properties and sand temperature were monitored continuously, while the sand water content was measured by gathering sand samples in the sand column at the conclusion of each experiment. Due to lack of time the experiments were limited to one pipe size and to more or less constant inlet airflow properties, even though it would have been preferable to model variations of these as well.

A plastic cover was used to stop the mass loss through the sand surface. This resulted in considerable temperature increase in the sand around the pipe and also a 50% lower irrigation yield. It was therefore concluded that the humid air transport from the pipe to the sand surface was the main contributor to the heat transfer in the sand. This illustrates that even though the gas phase are sometimes considered immobile in numerical simulation models, this is not a reasonable assumption under non-isothermal conditions.

The airflow temperature at the pipe inlet was shown to have the greatest impact on the irrigation yield for the given setup. This was explained by the fact that vapour content increases exponentially with increased temperature, while the airflow velocity and humidity correlation to vapour content is linear.

By increasing the duration of the experiments, it was seen that the irrigation rate declined with time because of the heat accumulation in the sand around the pipe. Nightly cooling by circulating low-temperature air through the pipe resolves this problem. This was however not done in the experiments.

2 Method

2.1 Experimental setup

To evaluate the Condensation Irrigation (CI) in practice, a small-scale CI plant was constructed inside a thermally insulated sand box, in which a humidified warm airflow was lead through a buried drainage pipe. For simplicity, only a thin cross-section of the sand was used, so that the system could be assumed to be two-dimensional.

The sand box was built 0.60 m high, 0.40 m across, and 0.20 m along the drainage pipe direction (Fig. 1). The pipe had a diameter of 50 mm, and the perforations in the pipe wall consisted of 6 evenly distributed 2 mm holes with a spacing of 20 mm. A thin cloth was wrapped around the pipe in the sand so that

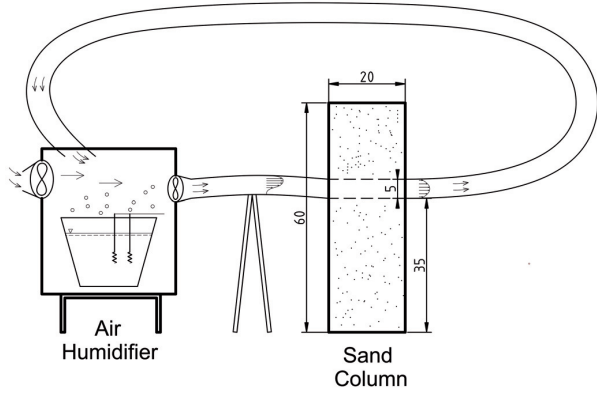


Figure 1: Experimental setup. The air was circulated through the humidifier and the sand column. Airflow properties were measured inside the pipe at the entrance and exit of the sand column.

sand would not enter the pipe through the perforations. The upper part of the pipe was located at a depth of 0.20 m from the sand surface.

The airflow was driven by a fan heater at the entrance to the humidifier, and another fan at the exit. Inside the humidifier the air was humidified and further heated by steam rising from a bucket with water heated by two electrical heaters. The air leaving the sand column was led back to the humidifier to create a semi-closed loop for the airflow. The setup is shown in Fig. 1.

The airflow properties and heat transfer in the sand column were monitored continuously, while the moisture profile in the sand was evaluated at the end of each test run. In Fig. 2 details of the setup are shown.

2.2 Determination of sand properties

If numerical simulations of the CI experiment are to be possible in the future, some necessary sand properties must be known or determined. The porosity, θ , of the sand was measured by stirring down a fixed bulk volume of sand, V_{sand} into a water-filled measuring cup. The volume increase of the water, ΔV_{water} , equaled the volume of the sand grains, so the porosity could be determined by:

$$\theta = \frac{V_{\text{sand}} - \Delta V_{\text{water}}}{V_{\text{sand}}}$$

The sand particle density, ρ_p (kg m^{-3}), was determined by dividing the weight of the sand volume used above and dividing it by the increase in water volume, ΔV_{water} . The dry bulk density, ρ_d (kg m^{-3}), is the product of the grain density and porosity. Relationships between the sand humidity and the capillarity and phase

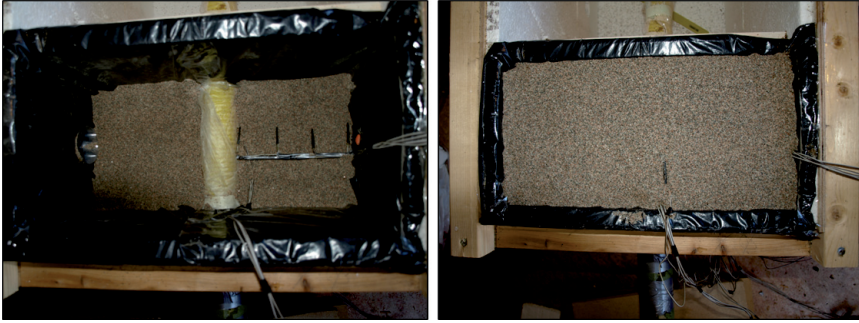


Figure 2: Sand column seen from above. In the left picture, the box is filled with sand to the lower part of the pipe. The four horizontal thermocouples and the thin cloth surrounding the pipe are visible. The right picture shows the sand column filled with dry sand before the start of the experiment. The topmost thermocouple is visible at the sand surface.

permeability were estimated by determining the so-called Soil Water Retention Curve (SWRC).

The SWRC was measured by placing a pillar of dry sand in a tray with a constant water level. The sand was wetted from below by capillary suction. When the soil water content was deemed constant, sand samples were collected at different heights and used for determining soil water saturation for the construction of the SWRC (Fig 3).

As a simplification, hysteresis was assumed to be negligible, so the SWRC resulting from the wetting experiment was set to be valid for both draining and wetting. This is a rough estimation and the wetting curve can be much lower than the draining curve. However, since condensation irrigation mainly is a wetting process, it is a reasonable approximation. According to [1] the SWRC can be generalized by the following expression:

$$h_c = \frac{1}{a} (S^{-1/m} - 1)^{1/n} \quad (1)$$

In Eq. (1), h_c (m) is the capillary rise above the free water surface, S is the water saturation degree, and a , n and $m = 1 - \frac{1}{n}$ are shape parameters for the function.

Measuring liquid and gas phase permeabilities in unsaturated soil is a very complicated and time-consuming process [5], so instead the parameters obtained from Eq. (1) can be used together with the theoretical functions developed by [2] for describing the relative permeabilities of the liquid and gas phases:

$$k_{rw} = S^{1/2} \cdot \left(1 - \left(1 - S^{1/n}\right)^n\right)^2$$

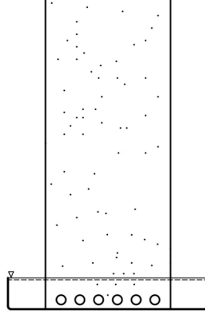


Figure 3: Sand filled transparent plastic cylinder standing in a tray with a constant water level. The holes in the bottom of the cylinder enable water to pass to and from the pillar of sand inside the cylinder.

$$k_{rg} = (1 - S)^{1/3} (1 - S^{1/n})^{2n}$$

The saturated hydraulic conductivity was measured using the same equipment as shown in Fig. 3 with a constant free water level above the sand column. From the time it took for 1 liter of water to infiltrate through the sand inside the cylinder while keeping the water level steady the Darcy velocity was obtained:

$$\nu = -k \frac{(\Delta h_{12})}{l_{12}}$$

where ν (m s^{-1}) is the water flux per unit area through a porous body, k (m s^{-1}) is the saturated hydraulic conductivity, Δh_{12} (m) is the height from the bottom of the sand to the water surface, and l_{12} is height of the sand column. The intrinsic permeability, k_i (m^2), of the sand was derived through the relationship

$$k_i = \frac{k \mu_w}{\rho_w g}$$

where μ_w (Pa s) is the dynamic viscosity of water and ρ_w (kg m^{-3}) is the water density.

2.3 Airflow measurements inside the drainage pipe

In the experiment, the air entering the drainage pipe was preheated and humidified by means of a fan heater and two electrical water heaters that heated water inside an open water bucket (Fig. 1). This apparatus could yield preset temperature and humidity for the airflow.

Upon entering and leaving the sand box, the airflow temperature was measured using type T thermocouples in the cross-sectional midpoint of the drainage pipe.

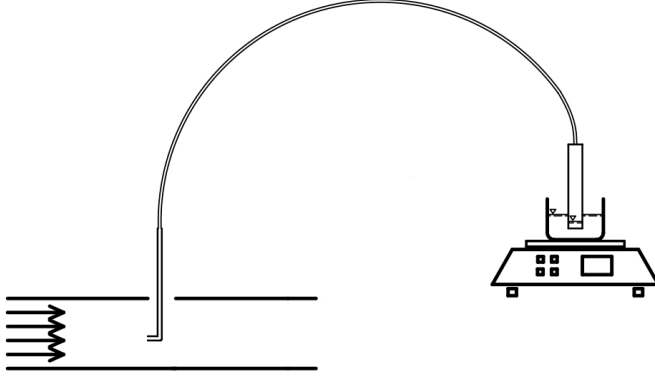


Figure 4: Equipment for measuring airflow pressure and velocity. A Prandtl tube was used to measure the total and static pressure inside the drainage pipe at the inlet and outlet to the sand column.

A wet temperature was also obtained from another thermocouple with a wet linen sock wrapped around it. From the difference in the wet and dry temperatures, T^w ($^{\circ}\text{C}$) and T^d ($^{\circ}\text{C}$), the airflow relative and absolute humidity, ϕ (%) and x , were estimated using Eqs. (2-3), in which the superscripts w and d denotes conditions at the wet and dry temperature, respectively:

$$x = \frac{0.622 \cdot p_{v,sat}^w}{p_{atm} + p_s - p_{v,sat}^w} \quad (2)$$

$$\phi^d = \frac{x \cdot (p_{atm} + p_s)}{p_{v,sat}^d (x + 0.622)} \quad (3)$$

In the Eqs. (2-3), p_{atm} (Pa) is the atmospheric pressure, p_s (Pa) is the static pressure inside the pipe, and $p_{v,sat}$ (Pa) is the saturated vapour pressure at the wet (w) or dry (d) temperature.

The total and static air pressures were measured using a Prandtl tube connected to a wider pipe partly immersed in a small water bowl. The bowl was placed on a laboratory scales to measure the airflow pressure working on the water inside the bowl, see Fig. 4. The scales had a precision of 0.01 g.

When aiming the nozzle of the Prandtl tube in flow direction, the pressure increase inside the tube was a result of the excess static air pressure inside the pipe. By placing the Prandtl tube in the opposite direction of the flow, both the static pressure and the flow-induced dynamic pressure were registered on the scales.

The air pressure in the Prandtl tube was calculated from the weight increase in the water bowl before and after the Prandtl tube was placed inside the drainage pipe:

$$H_g^{t,s} = \frac{\Delta m}{\rho_w A_c}$$

where $H_g^{t,s}$ (m) is the total (t) or static (s) pressure head of the humid airflow, Δm (kg) is the mass increase registered on the scales, and A_c (m²) is the cross-sectional area of the pipe leading the airflow into the water bowl. The dynamic pressure, being the difference between the total and the static pressure, allows for the humid air velocity, v_g (m s⁻¹), to be calculated:

$$v_g = \sqrt{\frac{2 \cdot \rho_w g \cdot (H_g^t - H_g^s)}{\rho_g}}$$

where H_g^t (m) and H_g^s (m) are the total and static pressures, and $\rho_g = \rho_a + \rho_v$ (kg m⁻³) is the humid air density.

By measuring the dynamic pressure at different locations in the pipe cross-section it was found that the maximum velocity was localized close to the top of the pipe, instead of in the middle. The causes of this anomaly were the sharp entrance from the humidifier to the pipe and the small bend of the pipe between the air humidifier and the sand column (Fig. 1).

A contraction coefficient, ψ , was for this reason multiplied to the pipe area to obtain the actual flow area [4]. At the outlet from the soil column the contraction coefficient was set to 1 due to the even flow profile at that location. The mass flux of water vapour, $\dot{m}_v$ (kg s⁻¹), at the in- and outlet of the drainage pipe is:

$$\dot{m}_v = v_g \rho_g \psi A \cdot \frac{x}{1+x} \quad (4)$$

where A (m²) is the cross-sectional area of the irrigation pipe. By assuming that the hourly measurements of the airflow properties represented average values over the whole hour, the potential irrigation yield, Δm_v (kg), was calculated as the difference between the inlet and outlet mass flux of vapour over the total time of operation in the experiment. The potential irrigation hence represented the total mass of water lost from the airflow between the pipe inlet and outlet.

While flowing through the sand column, some of the air left the pipe through the perforations, and transferred through the sand to the ambient air above the sand surface. The mass flux of air, $\dot{m}_a$ (kg s⁻¹), was calculated in a corresponding manner as in Eq. (4),

$$\dot{m}_a = v_g \rho_g \psi A \cdot \frac{1}{1+x} \quad (5)$$

and the total mass of air leaving the pipe through the sand, $\Delta \dot{m}_a$ (kg), was defined as the difference in the total mass flux of air entering and leaving the pipe over time of operation.

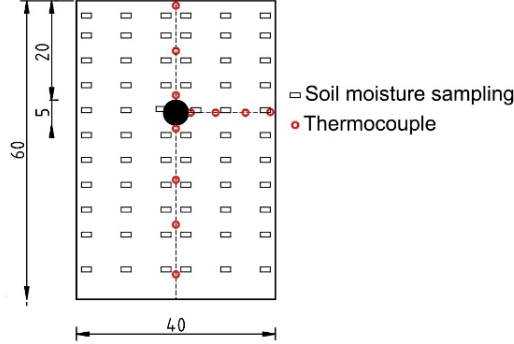


Figure 5: Location of the thermocouples and sand moisture sampling areas in the middle cross-section of the sand column

2.4 Measuring the soil temperature and moisture

During the experiments the soil temperature was recorded at 11 locations in the sand using type T thermocouples connected to a CR10 data-logger via a multiplexer. Measuring the water content of the sand during the test run was not deemed achievable since it is very difficult to extract sand samples at specific depths without disturbing the sand profile. Instead, sand samples were collected in the profile at the conclusion of each experiment. The samples were placed in small aluminum cups and directly weighted on the same scales used for the humid airflow measurements.

The sand samples from the first three experiments, were collected at three horizontal locations on eight elevations, resulting in 24 sand samples. In the following experiments, the sand sampling was increased to six horizontal locations and eleven elevations, i.e. 66 samples per experiment. Between each experiment the sand was replaced by new, dry sand from the same supply. Fig. 5 shows the location of the thermocouples in the sand and the locations for sand sample collection.

The collected sand samples were dried in 105 °C for at least three hours. The difference in weight before and after drying reveals the water content of each sample, which can be translated into water saturation, S , through the equation:

$$S = \frac{\rho_p (1 - \theta) (m_s^w - m_s^d)}{\rho_w \theta m_s^d}$$

where ρ_w (kg m⁻³) is the water density and m_s^w (kg) and m_s^d (kg) is the mass of the wet and dry sand sample, respectively. The net increase of sand moisture content, m_{sand} (kg), was estimated by first drawing up a sand moisture profile using the moisture contents found in the 24 respectively 66 soil samples, and then summing

up the contributions inside each area in the profile.

2.5 Calculations of the water balance in the sand

From the mass of dry air leaving the sand through the surface, the vapour mass lost through the sand surface could also be estimated. By assuming that the air in the pores at the surface of the sand has the same temperature as the bulk sand surface and that the relative humidity is $\varphi = 100\%$ (in a porous media $\varphi > 98\%$ even for very dry conditions), the vapour lost to the ambient, $m_{surface}$ (kg), could be calculated as:

$$m_{surface} = \Delta m_a \frac{0.622 \cdot p_{v,sat}(T_{surface})}{p_{atm} - p_{v,sat}(T_{surface})}$$

An error estimate of the mass balance of water, ε , could now be determined as

$$\varepsilon = \frac{\Delta m_v - m_{surface} - m_{sand}}{\Delta m_v}$$

in which m_{sand} (kg) is the accumulation of water in the sand, estimated from the sand samples gathered in the sand column.

2.6 Experiment execution

Eight experiment runs were carried out during a five month period. All runs comprised 8 hours of irrigation followed by 16 hours of recess, in which the airflow was stopped.

In the first experimental run (Exp 1) the sand surface was covered with a plastic foil so that no water could escape through surface evaporation. The purpose of using the plastic foil was to establish a mass balance between the accumulated soil moisture content in the sand and the calculated water loss in the pipe's airflow. The third experiment (Exp 3) was a repetition of the first experiment but with no plastic foil. In the second experimental run (Exp 2), the irrigation time was doubled to 10 days, but was otherwise the same as Exp 3.

It was concluded from the first three experiments that the humid airflow was not fully developed at the inlet to the sand column, and therefore the mass flux through the pipe was not calculated correctly. The pipe length between the humidifier and the sand column was therefore lengthened before continuing with the five subsequent experiments.

The longer pipe improved the flow profile of the air somewhat, but the fully developed flow was still not entirely achieved. A longer pipe had additional effects on the flow, such as reduced flow rate, and cooler air entering the sand column.

In the fourth and sixth runs (Exp 4 and Exp 6) irrigation was continued for three days, the fifth experiment (Exp 5) ran for 2 days, and the last two, Exp 7 and Exp 8, ran for 5 days each. The order of the experiments were selected randomly.

3 Results and Discussion

Before commencing with the experiments, relevant properties of the sand were measured. The porosity and particle density was estimated to $\theta = 0.40$ and $\rho_p = 2597 \text{ kg m}^{-3}$, respectively.

By curve fitting the data obtained from the capillary suction experiment against the Eq. (1), the shape parameters were determined to be $a = 14.8570 \text{ m}^{-1}$ and $n = 5.1021$ for the sand. The measured and fitted values for the SWRC are shown in Fig. 6.

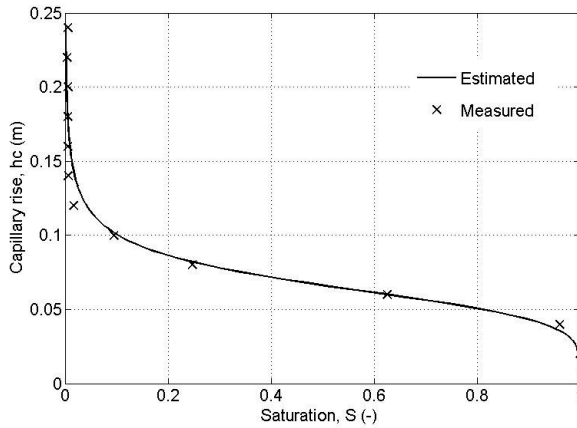


Figure 6: Estimated Soil Water Retention Curve under wetting conditions for the sand used in the experiments, expressed as $h_c=f(S)$ (m)

The hydraulic conductivity was $k = 8.19 \cdot 10^{-4} \text{ m s}^{-1}$. This value is high, even for a sand, because the sand particle size distribution was contained between 0.025 and 0.5 mm. The intrinsic permeability was $k_i = 8.38 \cdot 10^{-11} \text{ m}^2$.

3.1 Obtained Irrigation yield

The water amount found in the collected sand samples was translated into an irrigation amount by expressing the water gain in the soil as a corresponding precipitation on the surface (mm day^{-1}). This translation was done to get a clear

comparison of the irrigation yield between experiments. In Table 1 the irrigation levels and the inlet airflow properties are listed together with the duration of the experiments.

Table 1: Experiment duration and irrigation yield, $\bar{I}$ (mm day⁻¹), together with mean velocity, $\bar{v}$ (m s⁻¹), relative humidity, $\bar{\varphi}$ (%), and air temperature, $\bar{T}$ (°C) in the pipe at the inlet to the soil column. *Soil surface covered with plastic film

	Duration (days)	$\bar{I}$ (mm day ⁻¹)	$\bar{v}$ (m s ⁻¹)	$\bar{\varphi}$ (%)	$\bar{T}$ (°C)
Exp 1*	5	0.69	2.2	65	45.5
Exp 2	10	1.16	2.5	71	46.2
Exp 3	5	1.31	2.2	75	44.4
Exp 4	3	0.96	1.3	91	39.3
Exp 5	2	1.17	1.0	88	40.7
Exp 6	3	1.02	1.0	93	39.7
Exp 7	5	0.93	1.2	97	41.1
Exp 8	5	0.97	1.1	90	42.6

During the first three experiments a shorter pipe distance was used between the humidifier and the sand column, which meant that the incoming air had a higher temperature and velocity than in the following experiments. The resulting irrigation was therefore higher in these experiments than in the succeeding (Exp 1 excepted).

Previous theoretical studies have shown that the airflow temperature has the greatest impact on the irrigation yield: according to [3] a 20% increase in airflow temperature resulted in a 90% increase in irrigation. Increasing the inlet airflow velocity or humidity 20% resulted in about 10% and 30% more irrigation, respectively. These correlations fit quite well with the relations in these magnitudes between Exp 3 and Exp 7 or Exp 8.

3.2 Relations between mass and heat transfer

In the first experiment the soil surface was covered with a water tight plastic foil for the purpose of establishing an evaporation balance with other experiments. Instead, the surface plastic cover turned out to have completely other effects.

A comparison of the irrigation yield for Exp 1 and Exp 3 (Table 1), reveals that even though the two experiments had more or less the same inlet air temperature and velocity, the irrigation amount in Exp 1 was just over half of the irrigation obtained in Exp 3.

Hence, by sealing off the humid air path to the surface with the plastic foil, the gas transport in the soil was prevented. This greatly reduced the mass and heat transfer in the sand, leading to greater temperature gradients, and higher temperature around the pipe, and thus to lower irrigation yield. This explanation was supported by the moisture and temperature profiles for the two experiments, Figs. 7 and 8.

The moisture profiles in Fig. 7 clearly show the reduced water content in the sand in Exp 1 compared to Exp 3. The result of the reduced heat transfer in the sand is demonstrated in the temperature profiles (Fig. 8), in which the temperature above the pipe is 40°C in the sand with surface cover (Exp 1) and only 34°C in the experiment without (Exp 3).

The soil water in Exp 1 is transported upwards, away from the warm pipe wall, while in Exp 3, the water content is concentrated closer to the pipe. The reason why the water is not accumulated around the pipe in Exp 1 is due to the high temperature in the region around the pipe.

When gas transport is prohibited in the soil, the heat transfer must rely on the much slower process of heat conduction to even out the temperature differences. The density of the isotherms is a good indicator for the reduction in heat transfer. In the area of the highest moisture in Exp 1, the sand temperature is also found to be lower than the surrounding, since liquid water accumulates in the coolest region of a porous media.

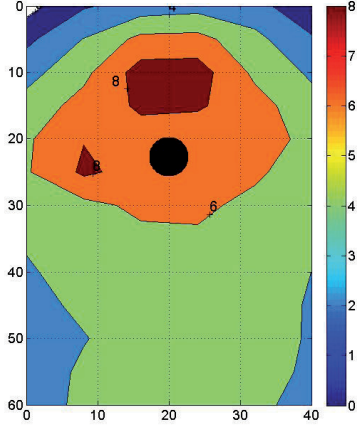
3.3 Water and heat propagation in the sand

The moisture and temperature profiles in the sand column after 2, 3, and 5 days of irrigation are shown in Figs. 9, 10, and 11. A mutual detail all of these moisture profiles possess is the asymmetrical accumulation of water to the right side of the pipe. Since this is true for all experiments, the explanation is most likely that the bar attaching the horizontal thermocouples had a higher water storage capacity than the sand.

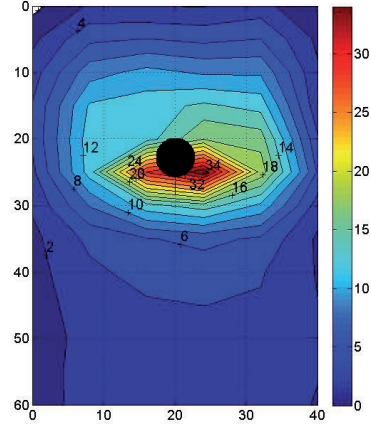
In the experiments with a three day irrigation period (Exp 4 and Exp 6), the inlet airflow temperature was lower than in the other experiments. As a result, the temperature profile in the sand in Fig. 10(b) shows a lower sand temperature around the pipe for these experiments than for the others. An additional effect of the low inlet air temperature was a relative reduction in the irrigation yield during the three days of operations (Exp 4 and Exp 6).

Since it was concluded from the first three experiments that gas movements in the sand was the dominant mode of heat transfer, the gas flow direction should coincide with the temperature gradient.

In all profiles, the temperature decreases radially out from the pipe with the smallest gradients in the upward direction, which indicates a better heat transfer above the pipe.

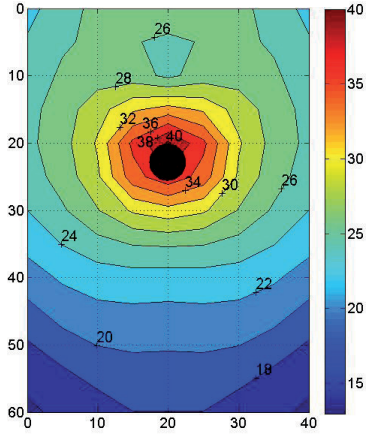


(a) Exp 1

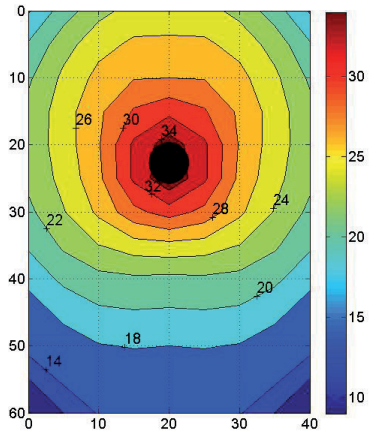


(b) Exp 3

Figure 7: Moisture profiles (kg m^{-3}) for Exp 1 and Exp 3.

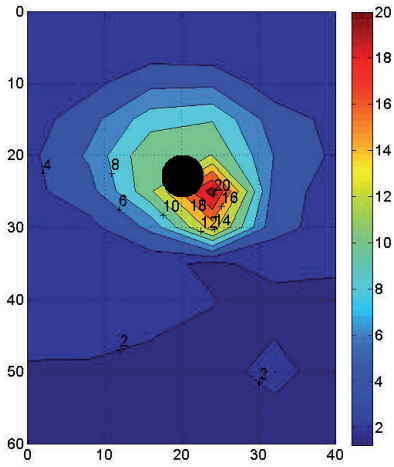


(a) Exp 1

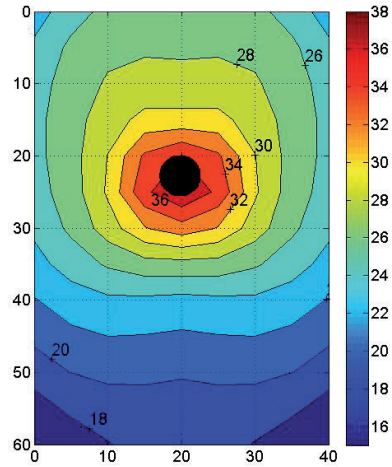


(b) Exp 3

Figure 8: Estimated temperature profiles ($^{\circ}\text{C}$) for Exp 1 and Exp 3

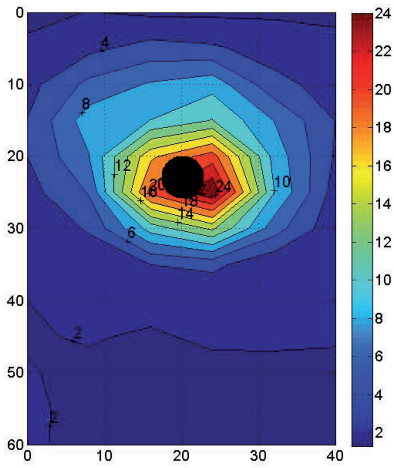


(a) Moisture content (kg m^{-3})

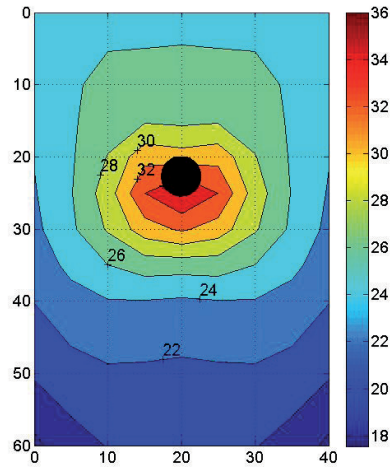


(b) Temperature profile ($^{\circ}\text{C}$)

Figure 9: Soil profiles after 2 days of irrigation (Exp 5)



(a) Moisture content (kg m^{-3})



(b) Temperature profile ($^{\circ}\text{C}$)

Figure 10: Soil profiles after 3 days of irrigation (average values from Exp 4 and Exp 6)

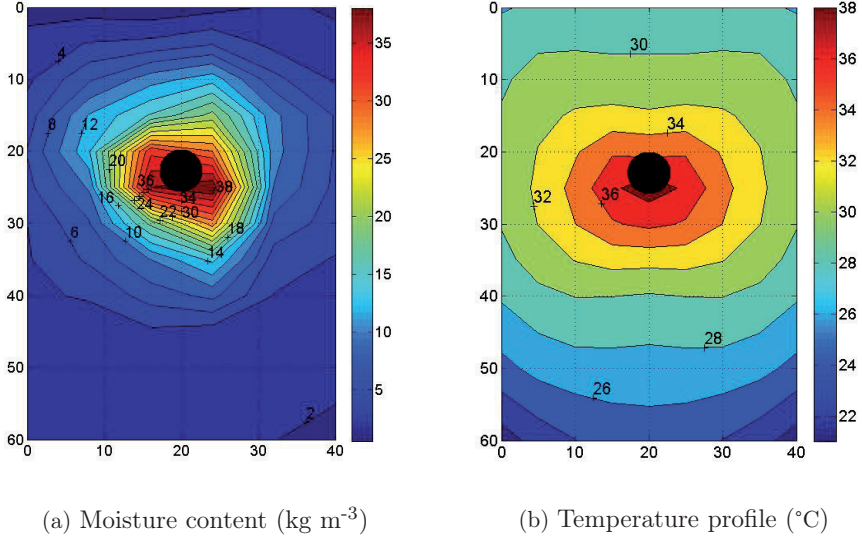


Figure 11: Soil profiles after 5 days of irrigation (average values from Exp 7 and Exp 8)

The gas thus mainly escapes the pipe in the radial direction, through the upper part of the pipe, and leaves through the sand surface. Due to gravity, the liquid water concentration is highest just below the pipe, where the gas advection is lower.

3.4 Irrigation decline with time

As previous theoretical simulations indicated [3], the daily irrigation yield declined each day of operation due to the increasing sand temperature. After some time, the daily irrigation should reach a steady yield, as the diurnal soil temperature variations reaches a steady state.

To see this more clearly, the irrigation yields are plotted for different durations in Fig. 12. The five last experiments are shown in the first three blocks, and thereafter Exp 2 and Exp 3 are displayed separately, due to their dissimilar inlet airflow properties.

Trend lines are included to visualize the irrigation decline towards a steady yield. The irrigation level after 3 days of operation are comparatively low due to the lower inlet air temperature in these experiments.

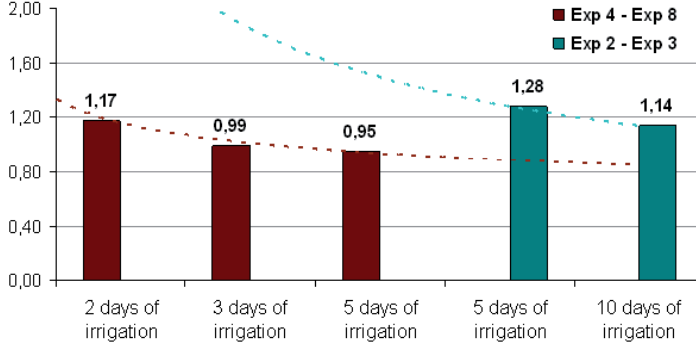


Figure 12: Daily irrigation yield (mm day^{-1}) compared to different duration of operation

3.5 Mass balance of water in the sand column

The applied contraction factor ψ was adjusted for each experiment so that most of the water entering the soil column through the pipe could be accounted for (Table 2). Due to the unstable flow, even for very small deviations in flow area, the reduction in vapour flux through the pipe was considerable, indicating that ψ was an important design parameter.

The actual irrigation yield in Table 1 was determined from the water content found in the sand samples at the conclusion of each experiment. In Table 2 the actual and potential irrigation levels, m_{sand} and Δm , are displayed together with the water loss through surface vapour flux, m_{surface} , and the mass balance error, ϵ , resulting from using the contraction factors ψ .

The calculated mass balance reveals that at best about half of the water leaving through the pipe perforations remain in the sand. The rest exit the sand through the sand surface. In Exp 1 the surface was covered with a plastic film to reduce vapour mass loss through the surface. Still, according to Table 2 it appears that some humid air managed to leak through the cover. Nevertheless, the required gas pressure for leaving the sand column was higher when passing through a much smaller opening.

In Table 2 it is also indicated that the relative amount of water leaving through the surface increases with time: for the experiments with short duration (Exp 4 to Exp 6), a larger fraction of water accumulates in the sand. With time, the accumulation of water in the sand, Δm , is overtaken by the surface loss of water, m_{surface} . The reason for this is that the sand in the beginning of an experiment is colder, which causes more vapour in the gas flow to condense in the sand near the pipe. As the sand is heated during the daily operation, the gas phase carries the vapour further and further away from the pipe, and the vapour content of the humid air leaving through the surface increases.

Table 2: Mass balance for the experiments. In the columns, Δm is the total mass of vapour lost between the inlet and outlet for the whole experiment duration, m_{surface} is the estimated vapour loss through the sand surface, and m_{sand} is the measured sand water content.

	Δm (g)	m_{surface} (g)	m_{sand} (g)	ψ (-)	ϵ (%)
Exp 1*	332	37	277	0.802	6%
Exp 2	2149	1149	927	0.812	3%
Exp 3	1085	523	520	0.848	4%
Exp 4	329	96	229	0.805	1%
Exp 5	246	52	187	0.936	3%
Exp 6	395	131	246	1.000	5%
Exp 7	728	336	370	0.925	3%
Exp 8	951	553	388	0.964	1%

*Soil surface covered with plastic film

The calculated error in Table 2 is a theoretical ratio of the estimated mass fractions in relation to the difference in vapour content in the pipe airflow upon entering and leaving the soil column. However, the only absolute value in Table 2 is the mass of water found in the sand, which also is associated with insecurities because of the assumption that the designated volumes around the sand sample had the same moisture content as that found in the sample.

The inlet airflow area had to be adjusted with the contraction factor ψ , since the flow profile in the pipe between the humidifier and the sand box was not developed throughout the whole cross-section, but concentrated in the upper part of the pipe. For this reason the mass of vapour entering the sand box with the airflow is uncertain.

Since each set of airflow measurements took about 20-40 minutes, the measurements were performed only once every hour. In the theoretical evaluation the airflow properties were assumed to be constant until the next measurement, even though this was never the case.

In the calculation of the humid air leaving through the sand surface, the air was assumed to have the same temperature as the sand surface. Since the vapour content increases exponentially with temperature, even a small change in temperature, may result in significant changes in the water balance. At the surface the sand temperature was measured right on top of the pipe inlet. This was probably the warmest region of the surface. On the other hand, the gas phase transported to the surface was the main contributor to heating the sand, which means that

the vapour was warmer than the average sand temperature. Hence, it is difficult to estimate the exact temperature of the gas phase at the surface.

4 Conclusion

Experiments on a small scale Condensation Irrigation (CI) system were made for determining the potential of cultivating crops by condensing humid air in underground horizontal drainage pipes. A 0.20 m long sand box was built reaching 0.40 m across by 0.60 m in height, with a 50 mm perforated horizontal pipe running through the section at a depth of 0.225 m. Due to the short distance along the drainage pipe (0.20 m), the heat and mass transfer in the sand profile could be assumed to be approximately two-dimensional.

By sealing off the sand surface from air and vapour flux, the significance of gas advection in the mass and heat transfer could be observed. With the surface covered, gas advection in the sand was hindered, which resulted in much higher sand temperature around the pipe, and much (53%) less irrigation yield than in the reference experiment. This indicated that the gas advection was a dominant factor in the heat transfer in the sand. Gas advection should hence never be neglected or assumed constant in simulations of heat and mass transfer in porous media.

Without any surface cover, the gas advection in the sand transported heat and humid air from the pipe wall to the surface of the sand. In doing so, the temperature around the pipe was greatly reduced, which increased the irrigation yield. In designing large scale CI plants, it is hence of importance to consider the porosity of the soil and apply proper tillage, so that gas advection is allowed in the soil.

The average irrigation yield in the experiments was 1.03 mm d^{-1} . Previous theoretical studies have shown that by increasing the temperature 20%, the irrigation yield increases almost 90%, mainly due to the exponential relation between the vapour partial pressure and temperature. Also, by increasing the velocity or humidity of the air, the irrigation level is increased. Hence, by increasing the inlet air temperature in Exp 2-Exp 8 from the average 42°C to 50°C , the irrigation should exceed 2 mm d^{-1} . Also by up-scaling the pipe diameter, the contact surface between the airflow and the cooler pipe wall could become larger, and the humid air flux to the sand would increase, which would increase the vapour condensation in both the pipe and the sand.

From examining the temperature profiles it was concluded that the gas transport occurred mainly from the upper half of the pipe's circumference. This conclusion was based on the fact that the lower half of the pipe was warmer, indicating less gas advection, and that the water saturation in the same region was higher due to percolated condensate from the pipe, which reduced the gas relative per-

meability.

By increasing the duration of the experiments, a reduction in the daily irrigation yield was observed. This reduction is a result of the gradually warmer sand around the pipe, which reduces the condensation rate in the pipe. As predicted, the reduction seemed to decline with time, so that the irrigation could reach a steady yield after some time. By cooling the sand at night, the irrigation yield would not decrease as much. In a field using CI, the cooling can e.g. be solved by circulating cool ambient air through the pipes during night, when no solar energy is available to power the irrigation.

Many of the insecurities in the results could have been abated by assuring a stable and uniform profile of the airflow inside the pipe. More repetitions of the same experiments would also have strengthened the relevance of the findings. By systematically varying the inlet airflow properties and the pipe diameter and depth, design parameters could more readily have been extracted from the results. Due to limitations in available time, this was not possible.

4.1 Acknowledgments

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Appendix

Discretization of mass and heat balance equations in the
simulation program CI2D

Discretization of the mass and heat balance equations in the simulation program CI2D

1. Temporal Discretization of Balance Equations

The balance equations of air, water, and heat transfer in unsaturated porous media is written:

$$a_P \frac{\partial H_g}{\partial t} + a_S \frac{\partial S_w}{\partial t} + a_T \frac{\partial T}{\partial t} + \nabla \bullet (\bar{i}_a + \rho_a \bar{v}_g) = f_a \quad (1.1)$$

$$\begin{aligned} a_H &= \theta (1 - S_w) \frac{\partial \rho_a}{\partial p_a} \rho_w g \\ a_S &= -\theta \rho_a \\ a_T &= \theta (1 - S_w) \left(\frac{\partial \rho_a}{\partial T} - \frac{\partial \rho_a}{\partial p_a} \frac{\partial p_v}{\partial T} \right) \end{aligned}$$

$$v_S \frac{\partial S_w}{\partial t} + v_T \frac{\partial T}{\partial t} + \nabla \bullet (\bar{i}_v + \rho_v \bar{v}_g) + \nabla \bullet (\rho_w \bar{v}_w) = f_w \quad (1.2)$$

$$\begin{aligned} v_S &= \theta (\rho_w - \rho_v) \\ v_T &= \theta \left(S_w \frac{\partial p_w}{\partial T} + (1 - S_w) \frac{\partial p_v}{\partial T} \right) \end{aligned}$$

$$e_P \frac{\partial H_g}{\partial t} + e_S \frac{\partial S_w}{\partial t} + e_T \frac{\partial T}{\partial t} + \nabla \bullet \mathbf{q} = f_e, \quad (1.3)$$

$$\begin{aligned}
e_H &= u_a a_H \\
e_S &= \theta (\rho_w h_w - \rho_v u_v - \rho_a u_a) \\
e_T &= \theta (1 - S_w) \left(\rho_v \frac{\partial u_v}{\partial T} + u_v \frac{\partial \rho_v}{\partial T} + \rho_a \frac{\partial u_a}{\partial T} + u_a \frac{\partial \rho_a}{\partial T} \right) \\
&\quad + \theta S_w \left(\rho_w \frac{\partial h_w}{\partial T} + h_w \frac{\partial \rho_w}{\partial T} \right) + (1 - \theta) \rho_p \frac{\partial h_p}{\partial T}
\end{aligned}$$

To analyze the balance equations numerically, the temporal and spatial terms must first be discretized. Due to the strong non-linearities in the equations, explicit approximations of the time derivatives are not appropriate, since the solution would be restricted to very small time-steps and low permeability soils in order to achieve convergence. Implicit solutions are therefore mostly used in modeling convective flow in porous media.

The choice of primary variables also influence the result of the numerical scheme. For example, using the water pressure head H_w (m) for estimating the water content over time, very large mass-balance errors might be produced because of the highly non-linear nature of the specific moisture capacity, $C_w = \frac{dS}{dH_c}$ [3]. There are also problems with obtaining solutions using pressure based formulations when the capillary gradients are small. On the other hand, if the water saturation is used as primary variable, the balance equation is no longer valid if the soil somewhere becomes water saturated, since the saturated flow cannot be measured in terms of saturation. For these reasons, a mixed formulation is used in the CI2D model, that should be free of these problems.

Celia et. al [3] proposed a water flow model based on the mixed formulation of the balance equation using the “modified Picard linearization”. The method is based on using a Picard iteration scheme of an implicit Euler approximation of the temporal terms in the balance equations together with a variable substitution of the saturation variable with capillary pressure. In doing so, both pressure and saturation are present in the solution. This method has been shown to conserve mass perfectly under isothermal conditions for both a single phase model, and for a combined air and water flow model [1, 2], and also to work well under nearly saturated to saturated conditions, where other temporal approximations may be unsuccessful.

In the CI2D model the modified Picard method is expanded to apply to non-isothermal situations. To show how the method has been used in CI2D, the air mass balance equation temporal discretization is derived below:

Firstly, an implicit Euler approximation is applied to the air balance equation:

$$\begin{aligned}
& a_P \frac{H_g^{t+1,n+1} - H_g^t}{\Delta t} + a_S \frac{S_w^{t+1,n+1} - S_w^t}{\Delta t} + a_T \frac{T^{t+1,n+1} - T^t}{\Delta t} \\
& - \nabla \bullet \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \\
& = f_a^{t+1,n}
\end{aligned}$$

The index $t+1$ represents the current time step and $n+1$ the current iteration. By adding and subtracting the former iteration value (index $t+1,n$) of the primary variables, the balance equation takes the form:

$$\begin{aligned}
& a_P \frac{\delta H_g}{\Delta t} + a_S \frac{\delta S_w}{\Delta t} + a_T \frac{\delta T}{\Delta t} - \nabla \bullet (\rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) \\
& = f_a^{t+1,n} - a_P \frac{H_g^{t+1,n} - H_g^t}{\Delta t} - a_S \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - a_T \frac{T^{t+1,n} - T^t}{\Delta t} \\
& + \nabla \bullet \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \\
& = R_A \tag{1.4}
\end{aligned}$$

$$\delta H_g = H_g^{t+1,n+1} - H_g^{t+1,n}$$

$$\delta S_w = S_w^{t+1,n+1} - S_w^{t+1,n}$$

$$\delta T = T^{t+1,n+1} - T^{t+1,n}$$

and R_A is the residual of the air balance equation, defined as the deviation of the former iteration from the correct estimate of the change in air mass over the time-step. As the iterations progress, R_A , δH_g , δS_w , and δT approach zero, and the solution thus converges.

To complete the temporal discretization of the air balance equation, the primary variable S_w is substituted by the water pressure head, H_w , through the relationship $H_c = H_w - H_g$, and by the specific moisture capacity function, C_w , defined as the slope of the Soil Water Retention Curve:

$$C_w \approx \frac{S_w^{t+1,n+1} - S_w^{t+1,n}}{H_c^{t+1,n+1} - H_c^{t+1,n}}$$

this is re-written as

$$\delta S_w = C_w (\delta H_w - \delta H_g)$$

Inserting this expression into Eq. (1.4) yields the final air balance equation:

$$\begin{aligned}
& a_P \frac{\delta H_g}{\Delta t} + a_S \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + a_T \frac{\delta T}{\Delta t} - \nabla \bullet (\rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) \\
& = f_a^{t+1,n} - a_P \frac{H_g^{t+1,n} - H_g^t}{\Delta t} - a_S \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - a_T \frac{T^{t+1,n} - T^t}{\Delta t} \\
& \quad + \nabla \bullet \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \\
& = R_A \quad (1.5)
\end{aligned}$$

Following the same procedure for the water and heat balance equation, these take the form:

$$\begin{aligned}
& v_S \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + v_T \frac{\delta T}{\Delta t} \\
& - \nabla \bullet (\rho_w^{t+1,n} k_w^{t+1,n} \nabla \delta H_w) - \nabla \bullet (\rho_v^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) \\
& = f_v^{t+1,n} - v_S \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - v_T \frac{T^{t+1,n} - T^t}{\Delta t} \\
& + \nabla \bullet \left(i_v^{t+1,n} + \rho_v^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \\
& \quad + \nabla \bullet \left(\rho_w^{t+1,n} k_w^{t+1,n} \left(\nabla H_w^{t+1,n} + \frac{\partial y}{\partial y} \right) \right) \\
& = R_V \quad (1.6)
\end{aligned}$$

and

$$\begin{aligned}
& e_P \frac{\delta H_g}{\Delta t} + e_S \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + e_T \frac{\delta T}{\Delta t} \\
& - \nabla \bullet (h_a^{t+1,n} \rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) - \nabla \bullet (h_v^{t+1,n} \rho_v^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) \\
& \quad - \nabla \bullet (h_w^{t+1,n} \rho_w^{t+1,n} k_w^{t+1,n} \nabla \delta H_w) - \nabla \bullet k_h^{t+1,n} \nabla \delta T \\
& = f_e^{t+1,n} - e_P \frac{H_g^{t+1,n} - H_g^t}{\Delta t} - e_S \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - e_T \frac{T^{t+1,n} - T^t}{\Delta t} \\
& \quad + \nabla \bullet h_a^{t+1,n} \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \\
& \quad + \nabla \bullet h_v^{t+1,n} \left(i_v^{t+1,n} + \rho_v^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \\
& \quad + \nabla \bullet h_w^{t+1,n} \left(\rho_w^{t+1,n} k_w^{t+1,n} \left(\nabla H_w^{t+1,n} + \frac{\partial y}{\partial y} \right) \right) + \nabla \bullet k_h^{t+1,n} \nabla T^{t+1,n} \\
& = R_E \quad (1.7)
\end{aligned}$$

To complete the adaptation of the mass and energy balance equations for numerical simulations, a spatial discretization must be applied to the flux terms. In the CI2D model, finite elements are used for this purpose.

2. Finite Element Formulation

For the spatial discretization of the balance Eqs. (1.5)-(1.7), Galerkin type finite elements were used because of the flexibility to describe irregular geometries and to include non-homogeneous properties. Due to the orthogonal geometry of the simulated two-dimensional Condensation Irrigation system, the model is, however, presently restricted to 2D rectangular elements. Although implementing triangular elements involves minor changes in the simulation program, analyzes of the validity of the program are much easier using rectangles.

Before discretization, the balance equations are expressed on the weak form which reduces the order of the equations from second order to first order differentials. In doing so, functions need only be continuous to one degree instead of two. Another result of the weak formulation is that the natural (Neumann) boundary fluxes are separated from the rest of the expression.

To obtain the weak form, the balance Eqs. (1.5)-(1.7) is first multiplied by an arbitrary test function $\omega(x, y) \neq 0$ and integrated over the area of a random element with thickness t^e . The flux terms are then integrated by parts so that the second derivative is assigned to the test function ω .

The weak formulation of the air balance equation becomes:

$$\begin{aligned}
 & t^e \int_{A^e} \omega \left(a_P \frac{\delta H_g}{\Delta t} + a_S \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + a_T \frac{\delta T}{\Delta t} \right) dA \\
 & - t^e \int_{A^e} \nabla \omega \bullet \left(\rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g \right) dA \\
 & = t^e \int_{A^e} \omega f_a dA \\
 & - t^e \int_{A^e} \omega \left(a_P \frac{H_g^{t+1,n} - H_g^t}{\Delta t} - a_S \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - a_T \frac{T^{t+1,n} - T^t}{\Delta t} \right) dA \\
 & - t^e \int_{A^e} \nabla \omega \bullet \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) dA \quad (2.1) \\
 & + t^e \left[\omega \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \right]_L \\
 & = R_A
 \end{aligned}$$

In the above equation, the term $t^e [\omega \rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g]_L$ has been neglected in the left side of the equation since δH_g approaches zero as the solution converges, which means that $\nabla \delta H_g \approx 0$. In the water and energy equations, the corresponding terms will also be neglected henceforth. The last term in Eq. (2.1) arise from the integration by parts and represents the surface integral of the flux over the element edges. The sum of this contribution is zero unless an external source or sink exists. This term, hence represents the natural (Neumann) boundary condition vector, and together with the f_a -vector term, constitutes the boundary vector F_A (kg s^{-1}):

$$F_A = t^e \int_{A^e} \omega f_a dA + t^e \left[\omega \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \right]_L$$

The weak formulation of the water balance equation:

$$\begin{aligned} & t^e \int_{A^e} \omega \left(v_S \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + v_T \frac{\delta T}{\Delta t} \right) dA \\ & - t^e \int_{A^e} \nabla \omega \bullet (\rho_w^{t+1,n} k_w^{t+1,n} \nabla \delta H_w) dA - t^e \int_{A^e} \nabla \omega \bullet (\rho_v^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) dA \\ & = -t^e \int_{A^e} \omega \left(v_S \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - v_T \frac{T^{t+1,n} - T^t}{\Delta t} \right) dA \\ & \quad - t^e \int_{A^e} \nabla \omega \bullet \left(\rho_w^{t+1,n} k_w^{t+1,n} \left(\nabla H_w^{t+1,n} + \frac{\partial y}{\partial y} \right) \right) dA \\ & \quad - t^e \int_{A^e} \nabla \omega \bullet \left(i_v^{t+1,n} + \rho_v^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) dA + F_V \\ & = R_V \quad (2.2) \end{aligned}$$

The corresponding boundary vector for water is F_V (kg s^{-1}) :

$$\begin{aligned} F_V &= t^e \int_{A^e} \omega f_v dA + t^e \left[\omega \rho_w^{t+1,n} k_w^{t+1,n} \left(\nabla H_w^{t+1,n} + \frac{\partial y}{\partial y} \right) \right]_L \\ & \quad + t^e \left[\omega \left(i_v^{t+1,n} + \rho_v^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \right]_L \end{aligned}$$

The weak form of the energy balance equation is:

$$\begin{aligned}
& t^e \int_{A^e} \omega \left(e_P \frac{\delta H_g}{\Delta t} + e_S \frac{C_w (\delta H_w - \delta H_g)}{\Delta t} + e_T \frac{\delta T}{\Delta t} \right) dA \\
& - t^e \int_{A^e} \nabla \omega \bullet (h_w^{t+1,n} \rho_w^{t+1,n} k_w^{t+1,n} \nabla \delta H_w) dA \\
& - t^e \int_{A^e} \nabla \omega \bullet (h_a^{t+1,n} \rho_a^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) dA \\
& - t^e \int_{A^e} \nabla \omega \bullet (h_v^{t+1,n} \rho_v^{t+1,n} k_g^{t+1,n} \nabla \delta H_g) dA \\
& = -t^e \int_{A^e} \omega \left(e_P \frac{H_g^{t+1,n} - H_g^t}{\Delta t} - e_S \frac{S_w^{t+1,n} - S_w^t}{\Delta t} - e_T \frac{T^{t+1,n} - T^t}{\Delta t} \right) dA \\
& - t^e \int_{A^e} \nabla \omega \bullet \left(h_a^{t+1,n} \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \right) dA \\
& - t^e \int_{A^e} \nabla \omega \bullet \left(h_v^{t+1,n} \left(i_v^{t+1,n} + \rho_v^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \right) dA \\
& - t^e \int_{A^e} \nabla \omega \bullet \left(h_w^{t+1,n} \rho_w^{t+1,n} k_w^{t+1,n} \left(\nabla H_w^{t+1,n} + \frac{\partial y}{\partial y} \right) \right) dA \\
& - t^e \int_{A^e} \nabla \omega \bullet k_h^{t+1,n} \nabla T^{t+1,n} dA + F_E \\
& = R_E \quad (2.3)
\end{aligned}$$

in which the energy boundary condition vector, F_E (W) is:

$$\begin{aligned}
F_E &= t^e \int_{A^e} \omega f_e dA + t^e [\omega k_h^{t+1,n} \nabla T^{t+1,n}]_L \\
&+ t^e \left[\omega h_a^{t+1,n} \left(i_a^{t+1,n} + \rho_a^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \right]_L \\
&+ t^e \left[\omega h_v^{t+1,n} \left(i_v^{t+1,n} + \rho_v^{t+1,n} k_g^{t+1,n} \left(\nabla H_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_w} \frac{\partial y}{\partial y} \right) \right) \right]_L \\
&+ t^e \left[\omega h_w^{t+1,n} \rho_w k_w^{t+1,n} \left(\nabla H_w^{t+1,n} + \frac{\partial y}{\partial y} \right) \right]_L
\end{aligned}$$

2.1. Spatial discretization of balance equations

Figure 2.1 shows a rectangular element with four internal nodes and the horizontal length L_x and the vertical height L_y . The internal node notation

from 1 to 4 is used when addressing a single element and is not the same as the global node notation, shown e.g in paper IV, section 2.3, Figure 2.1.

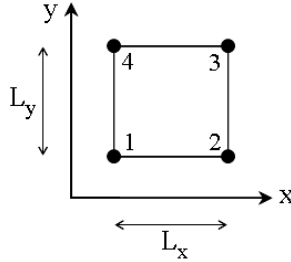


Figure 2.1. Internal node notation for a rectangular element

Assume that the value of a variable, $\alpha(t)$, can be estimated by a weighted mean value of the element's discrete nodal values, $\alpha_1(t)$, $\alpha_2(t)$, $\alpha_3(t)$, and $\alpha_4(t)$. If some arbitrary weighting parameters, w_1 - w_4 , are included for the nodal values, then

$$\alpha \approx \hat{\alpha} = w_1\alpha_1 + w_2\alpha_2 + w_3\alpha_3 + w_4\alpha_4 = \sum_{i=1}^4 w_i\alpha_i = \mathbf{w}\boldsymbol{\alpha} \quad (2.4)$$

in which the vector $\mathbf{w}$ comprising w_1 - w_4 is defined as

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix},$$

and the vector containing the node values of variable a is

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

The weighted estimate of the variable a must always have a value between the highest and lowest of the nodal values. To ensure this, the parameters in $\mathbf{w}$ are chosen so that

$$\sum_{i=1}^4 w_i = 1$$

and

$$0 \leq w_i \leq 1 \quad \text{for } i = 1, 2, 3, 4$$

Through Eq (2.4) the estimate $\hat{\alpha}$ is divided into the spatially dependent vector $\mathbf{w}$, consisting of the weighting parameters for the element nodes, and

the time dependent α -vector, consisting of the discrete values of a at the nodes. Thus, when differentiating $\hat{\alpha}$ over spatial coordinates, it is only the vector $\mathbf{w}$ that is concerned, since the nodal values of a are fixed in space. Reversely, when differencing over time, $\mathbf{w}$ is constant and the nodal values are evaluated. Hence:

$$\hat{\alpha}(x, y, t) = \mathbf{w}(x, y) \bullet \alpha(t) \Rightarrow \begin{cases} \frac{\partial \hat{\alpha}}{\partial x} = \frac{\partial \mathbf{w}(x, y)}{\partial x} \alpha(t) \\ \frac{\partial \hat{\alpha}}{\partial y} = \frac{\partial \mathbf{w}(x, y)}{\partial y} \alpha(t) \\ \frac{\partial \hat{\alpha}}{\partial t} = \frac{\partial \alpha(t)}{\partial t} \mathbf{w}(x, y) \end{cases} \quad (2.5)$$

If the weight functions w_1 - w_4 are assumed to be linear, so that all four nodes within the element in Figure 2.1 contribute equally to the estimation of the element value, they can be derived using Lagrange interpolating function:

$$\begin{aligned} w_1 &= \frac{(x-x_2)(y-y_4)}{A^e} \\ w_2 &= -\frac{(x-x_1)(y-y_3)}{A^e} \\ w_3 &= \frac{(x-x_4)(y-y_2)}{A^e} \\ w_4 &= -\frac{(x-x_3)(y-y_1)}{A^e} \end{aligned} \quad (2.6)$$

By inspection it is seen that each weight function has the value of 1 at its own node, and decreases linearly to 0 at the other nodes. Using the weight functions above, it is now possible to discretize the weak forms of the balance equations 2.1-2.3 by firstly substituting the arbitrary test function ω with the vector $\mathbf{w}^T(x, y)$, and then describing the affected variables according to Eq. (2.4)

$$\begin{aligned} T &\approx \hat{T}(x, y, t) = \sum_{i=1}^4 w_i(x, y) T_i(t) = \mathbf{w}^T \mathbf{T} \\ H_g &\approx \hat{H}_g(x, y, t) = \sum_{i=1}^4 w_i(x, y) p_{g,i}(t) = \mathbf{w}^T \mathbf{H}_g \\ H_w &\approx \hat{H}_w(x, y, t) = \sum_{i=1}^4 w_i(x, y) H_{w,i}(t) = \mathbf{w}^T \mathbf{H}_w \\ \chi_a &\approx \hat{\chi}_a(x, y, t) = \sum_{i=1}^4 w_i(x, y) \chi_{a,i}(t) = \mathbf{w}^T \chi_a \\ \chi_v &\approx \hat{\chi}_v(x, y, t) = \sum_{i=1}^4 w_i(x, y) \chi_{v,i}(t) = \mathbf{w}^T \chi_v \end{aligned}$$

According to Eq (2.5), the variable gradients in the weak formulations, Eq

(2.1)-(2.3), are now changed from ∇T to $\nabla \mathbf{w} \mathbf{T}$, etc. The balance equations take the form:

$$\begin{aligned}
& t^e \int_{A^e} \mathbf{w}^T \mathbf{w} dA \cdot \left(\mathbf{a}_p \frac{\delta \mathbf{H}_g}{\Delta t} + \mathbf{a}_s \frac{\mathbf{C}_w (\delta \mathbf{H}_w - \delta \mathbf{H}_g)}{\Delta t} + \mathbf{a}_T \frac{\delta \mathbf{T}}{\Delta t} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{k}_g^{t+1,n} \rho_a^{t+1,n} \nabla \mathbf{w} dA (\delta \mathbf{H}_g) \\
= & -t^e \int_{A^e} \mathbf{w}^T \mathbf{w} dA \cdot \left(\mathbf{a}_p \frac{\mathbf{H}_g^{t+1,n} - \mathbf{H}_g^t}{\Delta t} - \mathbf{a}_s \frac{\mathbf{S}_w^{t+1,n} - \mathbf{S}_w^t}{\Delta t} - \mathbf{a}_T \frac{\mathbf{T}^{t+1,n} - \mathbf{T}^t}{\Delta t} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{k}_g^{t+1,n} \rho_a^{t+1,n} \nabla \mathbf{w} dA \left(\mathbf{H}_g^{t+1,n} + \frac{\rho_a^{t+1,n}}{\rho_{w0}} \frac{\partial y}{\partial y} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{D}^{t+1,n} \nabla \mathbf{w} dA (\chi_a^{t+1,n}) + \mathbf{F}_A \\
& = \mathbf{R}_A
\end{aligned}$$

$$\begin{aligned}
& t^e \int_{A^e} \mathbf{w}^T \mathbf{w} dA \left(\mathbf{v}_s \frac{\mathbf{C}_w (\delta \mathbf{H}_w - \delta \mathbf{H}_g)}{\Delta t} + \mathbf{v}_T \frac{\delta \mathbf{T}}{\Delta t} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{k}_w^{t+1,n} \rho_w^{t+1,n} \nabla \mathbf{w} dA (\delta \mathbf{H}_w) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{k}_g^{t+1,n} \rho_v^{t+1,n} \nabla \mathbf{w} dA (\delta \mathbf{H}_g) \\
= & -t^e \int_{A^e} \mathbf{w}^T \mathbf{w} dA \left(\mathbf{v}_s \frac{\mathbf{S}_w^{t+1,n} - \mathbf{S}_w^t}{\Delta t} - \mathbf{v}_T \frac{\mathbf{T}^{t+1,n} - \mathbf{T}^t}{\Delta t} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{k}_w^{t+1,n} \rho_w^{t+1,n} \nabla \mathbf{w} dA \left(\mathbf{H}_w^{t+1,n} + \frac{\partial y}{\partial y} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{k}_g^{t+1,n} \rho_v^{t+1,n} \nabla \mathbf{w} dA \left(\mathbf{H}_g^{t+1,n} + \frac{\rho_v^{t+1,n}}{\rho_{w0}} \frac{\partial y}{\partial y} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{D}^{t+1,n} \nabla \mathbf{w} dA (\chi_v^{t+1,n}) + \mathbf{F}_V \\
& = \mathbf{R}_V
\end{aligned}$$

$$\begin{aligned}
& t^e \int_{A^e} \mathbf{w}^T \mathbf{w} dA \left(\mathbf{e}_P \frac{\delta \mathbf{H}_g}{\Delta t} + \mathbf{e}_S \frac{\mathbf{C}_w (\delta \mathbf{H}_w - \delta \mathbf{H}_g)}{\Delta t} + \mathbf{e}_T \frac{\delta \mathbf{T}}{\Delta t} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{h}_w^{t+1,n} \mathbf{k}_w^{t+1,n} \boldsymbol{\rho}_w^{t+1,n} \nabla \mathbf{w} dA (\delta \mathbf{H}_w) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{h}_a^{t+1,n} \mathbf{k}_g^{t+1,n} \boldsymbol{\rho}_a^{t+1,n} \nabla \mathbf{w} dA (\delta \mathbf{H}_g) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{h}_v^{t+1,n} \mathbf{k}_g^{t+1,n} \boldsymbol{\rho}_v^{t+1,n} \nabla \mathbf{w} dA (\delta \mathbf{H}_g) \\
& = -t^e \int_{A^e} \mathbf{w}^T \mathbf{w} dA \left(\mathbf{e}_P \frac{\mathbf{H}_g^{t+1,n} - \mathbf{H}_g^t}{\Delta t} - \mathbf{e}_S \frac{\mathbf{S}_w^{t+1,n} - \mathbf{S}_w^t}{\Delta t} - \mathbf{e}_T \frac{\mathbf{T}^{t+1,n} - \mathbf{T}^t}{\Delta t} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{h}_a^{t+1,n} \mathbf{k}_g^{t+1,n} \boldsymbol{\rho}_a^{t+1,n} \nabla \mathbf{w} dA \left(\mathbf{H}_g^{t+1,n} + \frac{\boldsymbol{\rho}_a^{t+1,n}}{\rho_{w0}} \frac{\partial y}{\partial y} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{h}_a^{t+1,n} \mathbf{D}^{t+1,n} \nabla \mathbf{w} dA (\chi_a^{t+1,n}) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{h}_v^{t+1,n} \mathbf{k}_g^{t+1,n} \boldsymbol{\rho}_v^{t+1,n} \nabla \mathbf{w} dA \left(\mathbf{H}_g^{t+1,n} + \frac{\boldsymbol{\rho}_v^{t+1,n}}{\rho_{w0}} \frac{\partial y}{\partial y} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{h}_v^{t+1,n} \mathbf{D}^{t+1,n} \nabla \mathbf{w} dA (\chi_v^{t+1,n}) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{h}_w^{t+1,n} \mathbf{k}_w^{t+1,n} \boldsymbol{\rho}_w^{t+1,n} \nabla \mathbf{w} dA \left(\mathbf{H}_w^{t+1,n} + \frac{\partial y}{\partial y} \right) \\
& - t^e \int_{A^e} \nabla \mathbf{w}^T \mathbf{k}_h^{t+1,n} \nabla \mathbf{w} dA (\mathbf{T}^{t+1,n}) + \mathbf{F}_E \\
& = \mathbf{R}_E
\end{aligned}$$

The boundary flux and source terms at each node are included in $\mathbf{F}_A$, $\mathbf{F}_V$, and $\mathbf{F}_V$. The weight function integrals associated with the flux and storage terms are commonly referred to as the stiffness matrix, $\mathbf{K}$, and the mass matrix, $\mathbf{M}$, respectively. Since the weight function $\mathbf{w}$ is a 1·4 row vector, it follows that both $\mathbf{K}$ and $\mathbf{M}$ are square 4·4 matrices.

2.2. Stiffness matrix

For a rectangular element with four internal nodes, such as shown in Figure 2.1, the stiffness matrix, $\mathbf{K}$, is a symmetrical 4·4 matrix. Using the internal

node notation from Figure 2.1, and the weight functions in Eq. (2.6), $\mathbf{K}$ is then written

$$\mathbf{K} = t^e \int_{A^e} \nabla \mathbf{w}^T \nabla \mathbf{w}$$

The gradient matrix for $\mathbf{w}$ is defined as:

$$\nabla \mathbf{w} = \begin{bmatrix} \frac{\partial w_1}{\partial x} & \frac{\partial w_2}{\partial x} & \frac{\partial w_3}{\partial x} & \frac{\partial w_4}{\partial x} \\ \frac{\partial w_1}{\partial y} & \frac{\partial w_2}{\partial y} & \frac{\partial w_3}{\partial y} & \frac{\partial w_4}{\partial y} \end{bmatrix}$$

Thus, when calculating K_{ij} for node i with respect to node j this becomes

$$K_{ij} = t^e \int_{A^e} \left(\frac{\partial w_i}{\partial x} \frac{\partial w_j}{\partial x} + \frac{\partial w_i}{\partial y} \frac{\partial w_j}{\partial y} \right) dA$$

For an arbitrary rectangular element this becomes :

$$\mathbf{K} = \frac{t^e}{6A^e} \begin{bmatrix} (2L_x^2 + 2L_y^2) & (L_x^2 - 2L_y^2) & (-L_x^2 - L_y^2) & (-2L_x^2 + L_y^2) \\ (L_x^2 - 2L_y^2) & (2L_x^2 + 2L_y^2) & (-2L_x^2 + L_y^2) & (-L_x^2 - L_y^2) \\ (-L_x^2 - L_y^2) & (-2L_x^2 + L_y^2) & (2L_x^2 + 2L_y^2) & (L_x^2 - 2L_y^2) \\ (-2L_x^2 + L_y^2) & (-L_x^2 - L_y^2) & (L_x^2 - 2L_y^2) & (2L_x^2 + 2L_y^2) \end{bmatrix}.$$

In the stiffness matrix, each row represents one node's relationship with itself and the other nodes, so at row i and column j , the weighted contribution from node j to node i is stated. Since the row sum of $\mathbf{K}$ is zero, the main diagonal of the matrix is hence the negative sum of the contributions from all the neighboring nodes.

Of note is that if the quotient between the element height, L_y , and width, L_x , (or vice versa) in the general $\mathbf{K}$ -matrix is less than $\sqrt{2}$, some of the off-diagonal entries in $\mathbf{K}$ becomes > 0 . This may in some cases give rise to oscillatory behavior in the numerical solution, so a constriction on element size ratio is set to $\sqrt{2}$.

2.3. Mass matrix

The consistent mass matrix is defined as:

$$\mathbf{M}_c = t^e \int_{A^e} \mathbf{w}^T \mathbf{w} dA$$

For a square-shaped element, such as in Figure 2.1, $\mathbf{M}_c$ becomes

$$\mathbf{M}_c = \frac{t^e A^e}{36} \begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}$$

However, when using the consistent form of the mass matrix, the storage terms are divided between the internal nodes of the element, which has proven to be a source of oscillations in numerical simulations, especially in situations with steep infiltration fronts. By lumping together the contributions from the nodes to the current node, a more monotone solution is achieved. In the CI2D model, row-sum lumping is applied, which means that the sum of each row in $\mathbf{M}_c$ is collected in the main diagonal. The lumped mass matrix, $\mathbf{M}$, for one element is hence:

$$\mathbf{M} = \frac{t^e A^e}{4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The term mass matrix is somewhat misleading, since the total sum of the matrix actually represents the element volume.

The matrices $\mathbf{K}$ and $\mathbf{M}$ and the column vectors $\mathbf{F}_A$, $\mathbf{F}_V$, and $\mathbf{F}_V$ are valid for one rectangular element. They are assembled for a complete geometry in the Matlab-code.

3. Upwind weighting of transport coefficients

The transport coefficients varies for each discrete location in the geometry. In slow and smooth transport processes, such as heat and mass diffusion, the mean value of the transport coefficients at the two discrete locations i and j usually works well to describe the transfer process between these points.

When describing advective flux in unsaturated porous media, the transport coefficient will often vary greatly over short distances. A mean value of the transport coefficient may then produce unstable or non-monotonic solutions to the transport problem, meaning that the solution may not converge, or that the spatial displacement may be oscillatory in nature. To avoid these problems, the mass and/or heat advective transport coefficients are weighted by their upwind values, i.e. at the node from which the flow occurs. For the liquid water advection, this is written [6, 5, 4]:

$$k_{w,ij} = \begin{cases} k_{w,i} & \text{if } (H_{w,i} - H_{w,j}) \geq 0 \\ k_{w,j} & \text{if } (H_{w,i} - H_{w,j}) < 0 \end{cases}.$$

Since all transport coefficients varies from node to node, the upwind and mean values of these are incorporated into the stiffness matrix to produce the estimated fluxes though the geometry.

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